Soil stiffness constitutive model parameters for geotechnical problems: A dilatometer testing approach

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ABSTRACT: Soil stiffness constitutive model parameters are required when investigating and/or modeling stress-related deformations in geotechnical problems. The seismic dilatometer test (SDMT) can be used to determine various stiffness related parameters. This paper discusses utilizing SDMT to determine the hardening plasticity parameters for the tangent modulus from primary oedometer loading (E_{oed}), secant modulus in drained triaxial test (E_{50}) , and unloading/reloading modulus (E_{ur}), along with the nonlinear small strain stiffness input parameters for the initial reference shear modulus (G_0) at very small strains and shear strain ($\gamma_{0.7}$) at which G_s=0.722G_0. The SDMT also provides evaluations of the soil strength and stress history for input into numerical simulations.

1 INTRODUCTION

Constitutive soil models consist of fundamental relations or mathematical definitions of a material's physical properties that describe how that geomaterial responds to external loading. An example of a commonly-used constitutive relation is Hooke's law relating stress and strain. Constitutive models form the basis of computational models analyzing stress and deformation. Numerical-method based analyses encountered in common geotechnical software utilize various constitutive soil models to characterize site conditions and predict soil response. Typical inputs to these models include stiffness parameters of subsurface soils that are defined using variants of Young's (elastic) modulus, E. Elastic modulus variants such as secant modulus in drained triaxial test (E_{50}), tangent modulus for primary oedometer loading (E_{oed}), as well as unloading/reloading modulus (E_{ur}), are often requested inputs in numerical analyses.

The purpose of this paper is to address the determination of site-specific soil stiffness parameters for finite element analyses utilizing in-situ testing methods, as opposed to performing rigorous laboratory testing. Small strain stiffness and nonlinear soil behavior can be determined utilizing SDMT. The fundamental shear modulus, G_0, is first determined for a soil profile using seismic shear wave velocity testing. Dilatometer testing of the subsurface then provides the basis to produce a G-γ modulus reduction curve for each representative soil type. Next, the G-γ modulus reduction curve is translated to an E-γ modulus reduction curve using elastic theory. The site specific E-γ modulus reduction curve along with data obtained from SDMT testing can then be used to determine modulus inputs for use in numerical simulations.

2 ELASTIC MODULI OF GEOMATERIALS

Soil stiffness is a complicated phenomenon. In the interest of simplicity, equivalent elastic soil stiffness parameters (elastic soil moduli) are defined as the ratio of stress along an axis to strain along an axis and often employed in soil characterization and analyses. Eq. 1 states the linear relationship between stress and strain using the proportionality factor of Young's (elastic) modulus, E:

$$\sigma = E\varepsilon \tag{1}$$

However, equivalent elastic modulus values that represent a simplification of true nonlinear soil behavior are complex. Elastic soil moduli values are dependent on internal and external factors that include soil state factors of particle organization (compaction), structural fabric, water content, stress history and cementation, along with loading factors that include the mean stress level in the soil, mean strain level in the ground, loading rate, loading cycles and drainage characteristics (Briaud 2001). Providing a singular modulus value for a given soil layer must include numerous considerations and adjustment factors and will also be a stress dependent value. Given the complexity of modulus definition, there are many available empirical modulus relationships. However, these relations do not adequately describe the in-situ soil state or address stress dependency. Site specific laboratory testing is required to fully consider in-situ soil state dependencies. Required triaxial and oedometer tests can be performed to more accurately determine stiffness values, but are most often time constrained and cost prohibitive.

Also of note is that many constitutive models use a reference stress at an effective confining stress of 1 atmosphere (100 kPa) when defining a modulus value and thus use the "ref" superscript.

Under applied shear stress, a given material will exhibit deformation and distortion. Shear modulus (or modulus of rigidity), G, is a measure relating shear stress to shear strain. For small strains, the shear modulus G is related to Young's Modulus, E, as follows through elasticity theory as applies to material properties:

$$E = 2 * G\,(1 + \nu) \quad (2)$$

where ν = Poisson's ratio, represents the elastic character of a material:

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} \quad (3)$$

where ε_{lat} = strain in the lateral direction and ε_{long} = strain in the longitudinal direction. The value of G_o, the fundamental small strain soil stiffness at initial loading is:

$$G_o = G_{max} = \rho_T * V_s^2 \quad (4)$$

where ρ_T = total soil mass density and V_s = shear wave velocity.

From Eq. 2 it can also be said for small strains, the fundamental elastic Young's modulus E_o can be represented as:

$$E_o = 2 * G_o\,(1 + \nu) \quad (5)$$

3 IN-SITU DETERMINATION OF SHEAR MODULUS

In-situ seismic testing methods can be used to discern G_o for various geomaterials. Geophysical crosshole seismic surveys, downhole surveys, suspension logging, seismic dilatometer (SDMT), seismic piezocone penetration tests (SCPTu), and spectral analysis of surface waves (SASW/MASW) are common in-situ tests that measure the profile of shear wave velocity (V_s) with depth.

Laboratory tests to determine G_o for soils are also available (e.g., bender elements, resonant column), yet direct measurement by field tests such as the SDMT are preferable to reliably determine V_s since lab results can be significantly affected by sample disturbance and stress relaxation issues, as well as greater expense in money and time.

4 NONLINEAR SOIL STIFFNESS AND STIFFNESS REDUCTION

When applied stresses produce strain levels, γ, that exceed the small strain limit of $\gamma_s < 10^{-6}$, the corresponding values of shear modulus G must utilize shear modulus reduction curves to obtain the appropriate value of G since the modulus softens with increased loading. The resulting hyperbolic behavior demonstrates the nonlinear behavior of soils subject to increased stress.

Hyperbolic G-γ modulus reduction curves follow the typical behavior indicated by Fig. 1 (Hardin & Drnevich, 1972). The representative Hardin-Drnevich curve is defined as:

$$\frac{G}{G_o} = \frac{1}{1 + \left(\frac{\gamma}{\gamma_{ref}}\right)} \quad (6)$$

where γ_{ref} = reference strain as detailed later.

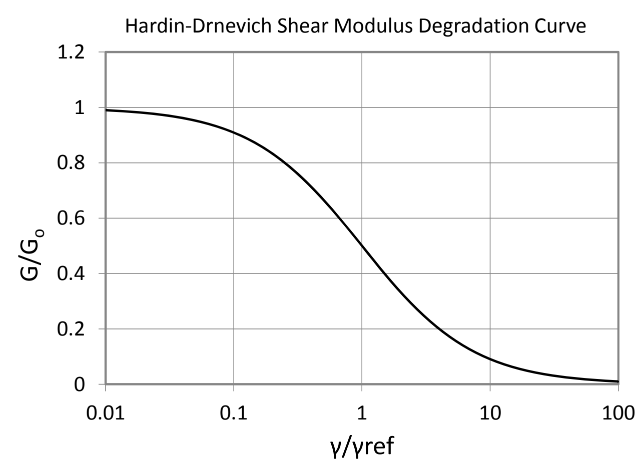

Fig. 1. Shear modulus reduction curve (after Hardin and Drnevich 1972)

The general Hardin-Drnevich relation has been further modified to include scaling factors in order to achieve a best fit hyperbolic model of modulus reduction for various soil types based on laboratory testing. The scaling factors are seen in the inclusion of a power exponent (α) as shown in Eq. 7 (Vardenega & Bolton 2011), or alternatively, a multiplicative factor (a) as shown in Eq. 8 (Santos & Correia, 2003).

$$\frac{G}{G_o} = \frac{1}{1 + \left(\frac{\gamma}{\gamma_{ref}}\right)^{\alpha}} \tag{7}$$

$$\frac{G}{G_o} = \frac{1}{1 + a\left(\frac{\gamma}{\gamma_{ref}}\right)} \tag{8}$$

The modified Hardin-Drnevich G-γ expression curves are the basis for modulus reduction curves used in several numerical modeling suites. The value of $a = 0.385$ in Eq. 8 after Santos and Correia (2003) is common to many hardening soil models (Benz, 2007).

The validation of the G-γ degradation curves from Santos & Correia (2003) utilized both sand and clay sites where $a = 0.385$ was found to be a best fit. The G-γ reduction curves from Vardenega & Bolton (2011) are based on a database of 20 silty and clayey soils sites where α = 0.74 was found to be a best fit. Fig. 2 shows the best fit reduction curves from the two approaches. As shown in Fig. 2, the use of the power α model of Eq. 7 gives a more rapid reduction of stiffness than does the multiplicative factor a model of Eq. 8.

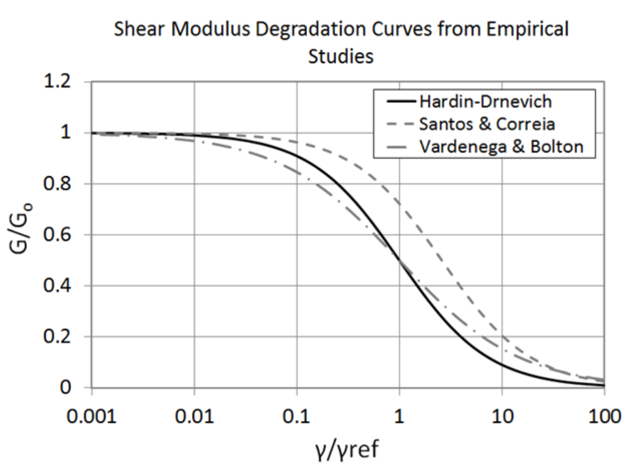

Fig. 2. Reduction curves from fitted experimental data studies

5 REFERENCE STRAIN

A commonality to both of the aforementioned G-γ modulus reduction relations is the normalization of shear strain. The original Hardin-Drnevich formulation for hyperbolic stiffness reduction is given in terms of a normalized reference shear strain. The reference shear strain in the Hardin-Drnevich equation is equal to the maximum shear stress or failure stress τ_{max} divided by the fundamental shear modulus G_0, thus:

$$\gamma_{ref} = \frac{\tau_{max}}{G_0} \tag{9}$$

The Vardenega and Bolton relation given in Eq. 7 is formulated after Dardeneli (2001) and Zhang et. al. (2005) with the power exponent α based on the definition that the secant shear stiffness reduces to half its initial maximum value ($G/G_0 = 0.5$) when $\gamma = \gamma_{ref}$. The determination of γ_{ref} is provided by Vardenega and Bolton (2011) as a function of plasticity index, liquid limit, plastic limit or void ratio, with reasonable agreement for the silts and clays studied.

The Santos and Correia (2003) relation given in Eq. 8 uses the volumetric threshold shear strain $\gamma_t^v = \gamma_{ref} = \gamma_{0.7}$ as a reference strain. The volumetric threshold shear strain indicates the strain limit at which irreversible change occurs in the soil structure (Vucetic & Dobry, 1991). This strain limit has been given as the strain level at which the ratio of G/G_0 is equal to 0.722 corresponding to a 27.8% reduction in modulus (Vucetic, 1994). For this reason, the reference value for volumetric shear strain is abbreviated as $\gamma_{0.7}$. Most numerical modeling software include the values of G_0 and $\gamma_{0.7}$ as inputs to define the stiffness reduction relationship for various geomaterials. The reason given for normalization using the volumetric threshold shear strain $\gamma_{0.7}$ is that the stiffness reduction becomes less prone to errors. (Benz, 2007).

The normalization of shear strain given in Eq. 9 has been shown to remove some limitations in characterizing behavior using different test modes, such as triaxial and direct shear testing (Drnevich, 1981). It has been shown when using a normalized reference shear strain as a function of τ_{max} as shown in Eq. 9, stress-strain curves for undrained and drained tests on like samples are approximately the same.

Studies of the deformation parameters of Sydney kaolin using stress history and normalized soil engineering properties (SHANSEP) methods (Poulos, 1974 & 1978) address the determination of drained and undrained deformation parameters from triaxial testing. Drained secant elastic moduli (E′), undrained secant elastic moduli (E_u), drained Poisson's ratio (ν′) values and undrained Poisson's ratio (ν_u) values were examined when utilizing various modes of triaxial testing at varying mobilized strengths. Testing modes included constant rate drained loading, constant rate undrained loading, one-stage dead loading under anisotropic initial stress conditions with drained conditions and two-stage dead loading under anisotropic initials stress conditions with undrained conditions followed by drained conditions. Specimens were studied with respect to normalization by the initial effective vertical strain and overconsolidation ratio (OCR).

Findings of the study indicated that undrained and drained deformation parameters showed good agreement when normalized using SHANSEP and are shown in Figs. 3 and 4. The study also showed that triaxial testing using isotropic initial state stresses yielded significantly larger values of E_u and E′ than did anisotropic initial state tests and that accurately determining a value of E_u from triaxial testing was difficult.

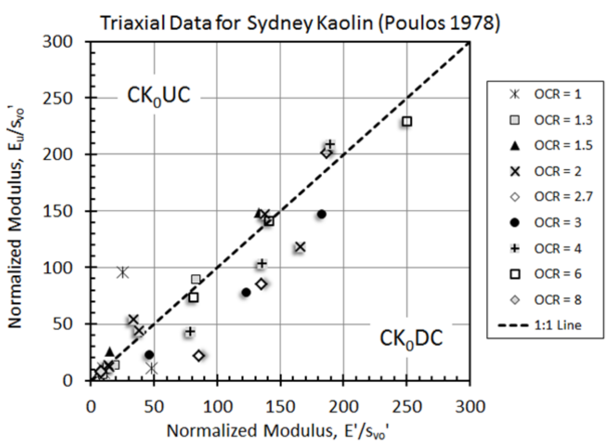

Fig. 3. Normalized drained and undrained secant modulus value with respect to OCR (Poulos, 1978)

6 DMT-BASED MODULUS REDUCTION CURVES

The seismic flat plate dilatometer can be used to generate a complete G-γ modulus reduction curve for a representative soil type. Hybrid geotechnical-geophysical tests such as the SDMT provide a means to economically and expediently evaluate G_0 profiles in various geomaterials. As shown by Amoroso (2012), after determining G_0 at small strains, an additional point should be determined on the stress-strain curve in order to develop the G-γ response via the modified hyperbolic equations. The additional point on the modulus reduction curve can

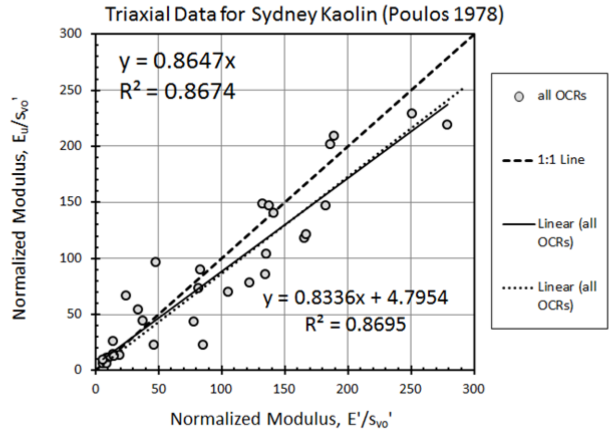

Fig. 4. Normalized drained and undrained secant modulus value with regression (Poulos, 1978)

be determined using SDMT testing via G_{DMT}, the shear modulus at working strains. The working strain G_{DMT} can be determined from SDMT testing using elastic theory as follows:

$$G_{DMT} = \frac{M_{DMT}}{2(1-\nu)/(1-2\nu)} \tag{10}$$

where M_{DMT} is the constrained modulus from SDMT testing corresponding to a working strain. The assumption that M_{DMT} represents a working strain value of the constrained modulus is based on previous studies and predictions including those by Monaco et. al. (2006) and Marchetti et. al. (2008).

It should be noted, however, that when using Eq. (10), the value of Poisson's ratio, ν, cannot be equal to 0.5, else the value of G_{DMT} is undefined.

Once the value of working strain shear modulus G_{DMT} is determined, the working shear strain γ_{DMT} corresponding with G_{DMT} must be determined in order to construct the G-γ modulus degradation curve. Previous studies as detailed by Amoroso et al. (2012) have estimated typical ranges of γ_{DMT} in different soil types using stiffness decay curves that are backfigured from the observed field behavior under full-scale loading, obtained by cyclic and dynamic laboratory tests or reconstructed by the combined use of different in situ and laboratory techniques. Typical ranges of γ_{DMT} are approximated as $\gamma_{DMT} \approx 0.01$–$0.45$ % in sand, $\gamma_{DMT} \approx 0.1$–1.9 % in silt and clay, $\gamma_{DMT} > 2$ % in soft clay (Amoroso et. al., 2012).

A G-γ modulus degradation curve determined using in-situ SDMT testing is shown in Fig. 5 for a stiff clay soil at a research site in Perth, Australia (Fahey et. al., 2003). The fundamental small strain soil stiffness, G_o, was determined from seismic shear wave velocity measurements. The working strain shear modulus, G_{DMT}, was then determined and the G-γ modulus degradation curve was constructed using Eqs. 6, 7 and 8 along with the soil parameters from SMDT testing found in Table 1 (Amoroso et. al., 2012).

Table 1. Modulus Parameters Perth, Australia

Shear wave velocity, V_s	334 m/s
Fundamental shear wave modulus, G_o	212 MPa
Constrained modulus from DMT, M_{DMT}	52 MPa
Poisson's ratio, ν	0.30
Working strain shear modulus, G_{DMT}	15 MPa
Normalized Working strain shear modulus, G_{DMT}/G_o	0.07
Working shear strain, γ_{DMT}	1.5 %
Maximum shear stress, τ_{max}	225 kPa

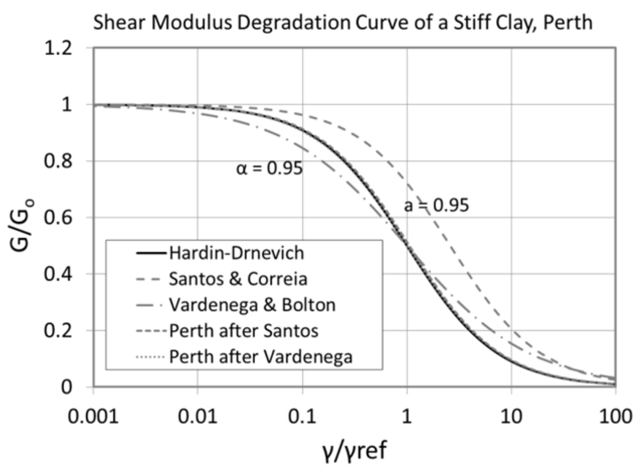

Fig. 5. G-γ modulus degradation curves utilizing in-situ testing

7 CONSTITUTIVE MODEL APPLICATIONS: ELASTIC MODULUS VALUES

Constitutive models that analyze stress and deformation in geotechnical analyses require stiffness parameters of geomaterials as inputs.

Typical inputs to these models include stiffness parameters of subsurface soils that are defined using variants of Young's (elastic) modulus, E. Once the G-γ modulus degradation curve is determined using in-situ testing, a corresponding E-γ modulus degradation curve can be constructed using Hooke's law and Eq. 2. The associated E-γ modulus degradation curve corresponding to the G-γ modulus degradation curve produced in Fig. 5 is shown in Fig. 6.

The constrained (tangent) modulus is determined through SDMT testing and is referred to in this paper as M_{DMT}. For normally consolidated soils,

$$M_{DMT} = \frac{1}{m_v} = E_{oed} = E_{DDMT} \tag{11}$$

where E_{DDMT} is defined as the modulus obtained from DMT testing used to define the E-γ degradation curve.

The secant modulus in drained triaxial testing at 50 percent strength E_{50} can also be determined using values obtained from SDMT testing. Where according to Vermeer (2001),

$$E_{50} \cong M_{DMT} \tag{12}$$

From the stiffness degradation curve and using Eq. 5, 10 and 11, the constrained tangent modulus E_{oed} (E_{DDMT}), can be plotted on the curve as shown in Fig. 7. The working shear strain γ_{DMT} must also be determined as the abscissa to the value of E_{DDMT} (or G_{DMT}) as previously discussed.

The unloading/reloading modulus in the drained/undrained triaxial test, $E_{ur,}$ cannot readily be determined using data obtained from DMT testing and must be calculated using accepted relationships if not using laboratory testing such as that given by Vermeer (2001),

$$E_{ur} \cong 4E_{50} \tag{13}$$

One will note that when viewing the stiffness degradation curve, E_{50} is the smallest of the modulus values discussed. Most numerical programs maintain an elastic stiffness cutoff at E_{ur} (corresponding to G_{ur}), where hardening plasticity accounts for further stiffness reductions.

Advanced hardening models include the values of G_o and $\gamma_{0.7}$ as inputs to define the nonlinearity and small strain stiffness relationships for various geomaterials. Once G_o is determined from seismic shear wave velocity testing, the stiffness degradation curve can be used to define $\gamma_{0.7}$ or various correlations have been developed to define the value. This strain limit has been given as the strain level at which the ratio of G/G_o is equal to 0.722, thus using the degradation curve and the ordinate of 0.722, the corresponding abscissa will yield $\gamma_{0.7}$.

8 CONSTITUTIVE MODEL APPLICATIONS: STRENGTH AND STRESS HISTORY

Although the focus of this paper was on required stiffness parameter inputs for constitutive models, strength and stress history input values for geomaterials are also needed in a numerical model. DMT testing also provides strength evaluations of friction angle (φ') and undrained shear strength (c_u) and the stress history parameters concerning lateral stresses (K_o), overconsolidation ratio (OCR), and soil unit weight (γ). Additional strength definitions such as dilatancy angle (ψ) can be determined indirectly, using existing correlations.

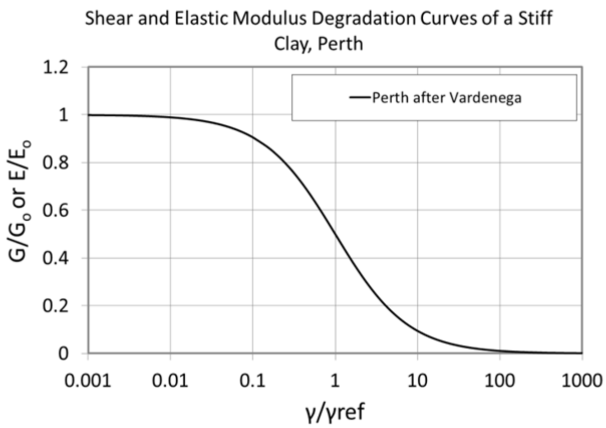

Fig. 6. E-γ modulus degradation curve constructed using G-γ modulus degradation curve

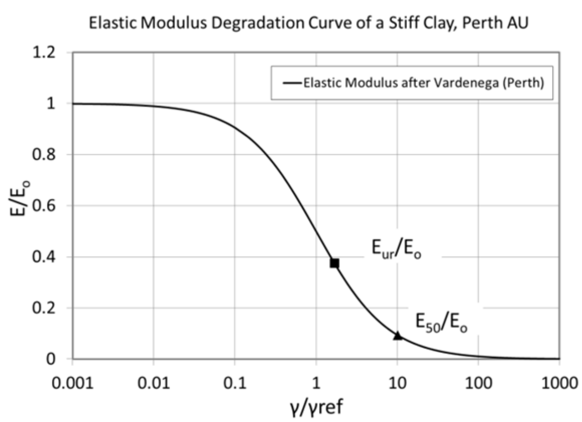

Fig. 7. E-γ modulus degradation curve

Marchetti's relationships for friction angle (φ') for sands, undrained shear strength of clays (c_u) lateral at rest pressure (K_o), and OCR are typically used values as defined as functions of horizontal stress index (K_D) as follows (Marchetti 1997):

$$\varphi = 28 + 14.61 \log K_D - 2.1 \log^2 K_D \quad (14)$$

$$c_u = 0.22\, \sigma'_{vo}\, (0.5\, K_D)^{\,1.25} \quad (15)$$

$$K_o = \left(\frac{K_D}{1.5}\right)^{0.47} - 0.6 \quad (16)$$

$$\mathrm{OCR} = (0.5\, K_D)^{1.56} \quad (17)$$

An alternative relationship for friction angle (φ') is as follows (Campanella & Robertson, 1991):

$$\varphi' = 37.3 \left(\frac{K_D - 0.8}{K_D + 0.8}\right)^{0.47} \quad (18)$$

An alternative relationship for undrained shear strength (c_u) based on the corrected first pressure reading (p_o) and hydrostatic porewater pressure (u_o) is as follows (Schmertmann, 1981):

$$c_u = s_u = \frac{p_o - u_o}{10} \quad (19)$$

Total unit weight can be estimated from soil type and relative density or from SDMT as follows (Mayne et al., 2002) where γ_w is unit weight of water, E_D is dilatometer modulus, p_a is atmospheric pressure and I_D is material index.

$$\gamma_T = 1.12\, \gamma_W \left(\frac{E_D}{p_a}\right)^{0.1} I_D^{\,-0.05} \quad (20)$$

Values for dilatancy angle (ψ) cannot be directly determined from SDMT testing. A correlation between in-situ state parameter (ξ_0) based on DMT testing is provided where a negative value indicates soils denser than critical state and a positive value indicates soils looser than the critical state (Yu, 2004):

$$\xi_0 = -0.002 \left(\frac{K_D}{K_0}\right)^2 + 0.015 \left(\frac{K_D}{K_0}\right) + 0.0026 \quad (21)$$

Granular soils have dilatancy angles that relate the volume change of void ratio as follows based on relative density, index, D_r and relative dilatancy index, I_r (Bolton, 1986):

$$I_r = 5D_r - 1 \qquad 0 < I_r < 4 \quad (22)$$

Plane strain conditions: ψ=$6.25 I_r$

Triaxial conditions: ψ=$3.75 I_r$

Cohesive soils may exhibit no dilatancy at all and can be generally classified as given in Table 2 (Obrzud & Truty, 2010):

Table 2. Dilatancy of Cohesive Soils

Normally consolidated or lightly consolidated Soil	Ψ = 0°
Overconsolidated soil	Ψ = ϕ'/3
Heavily overconsolidated soil	Ψ = ϕ'/6

9 CONCLUSIONS

Site-specific soil stiffness parameters were examined for use in soil modeling applications, particularly finite element simulations. Seismic shear wave velocity testing provides the fundamental small-strain shear modulus, G_0, for a soil profile. Dilatometer testing of the soils was used to produce a G-γ decay curve of a representative geomaterial by establishing the working strain shear modulus G_{DMT} corresponding to a working shear strain γ_{DMT}. The G-γ reduction curve was then translated into a E-γ decay curve using the theory of elasticity.

The use of a reference strain or threshold strain according to Hardin-Drnevich (1972) may prove useful if the reduction relationship can be easily implemented in numerical models. Utilizing a reference shear strain according to work by Drnevich (1979, 1981) is well suited to in-situ geomaterial characterization as drainage conditions are accounted for. Response of undrained and drained kaolin soils under triaxial testing as performed by Poulos provide additional compatability of drained and undrained conditions when measuring modulus values.

Available advanced hardening soil models for numerical modeling utilize $\gamma_t^v = \gamma_{0.7}$ at this time. More understanding of the applicability of the volumetric cyclic threshold shear strain and its implications within static loading cases may be warranted.

Evaluation of soil strength and stress history utilizing SDMT was also addressed. In summary, constitutive model parameters can be well defined for various geomaterials when utilizing in-situ testing methods, specifically SDMT testing, which can be performed both on land and offshore. Further investigation into working shear strain values, γ_{DMT}, corresponding with working shear modulus, G_{DMT}, could also benefit in-situ stiffness characterization.

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