Principles of Foundation Engineering

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Chapter 3

Shallow Foundations: Ultimate Bearing Capacity

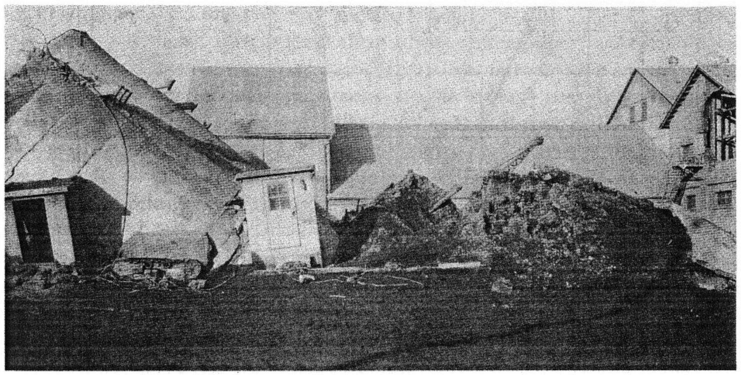
Unit Weights

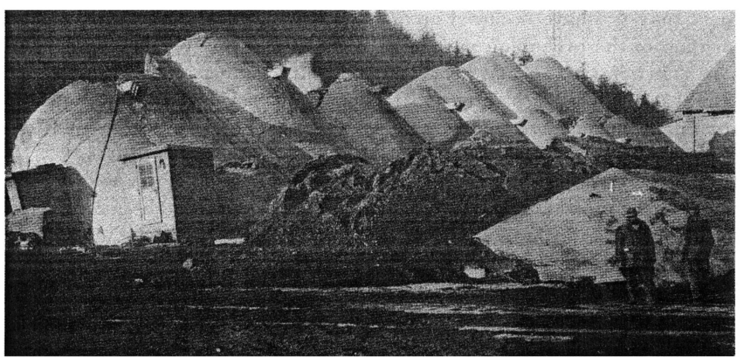
γ_w – unit weight of water
γ_d – dry unit weight (no moisture, just air)
γ_m – moist unit weight (has moisture & air)
γ_s – saturated unit weight (all moisture, no air)
γ' – bouyant unit weight = $\gamma_s - \gamma_w$

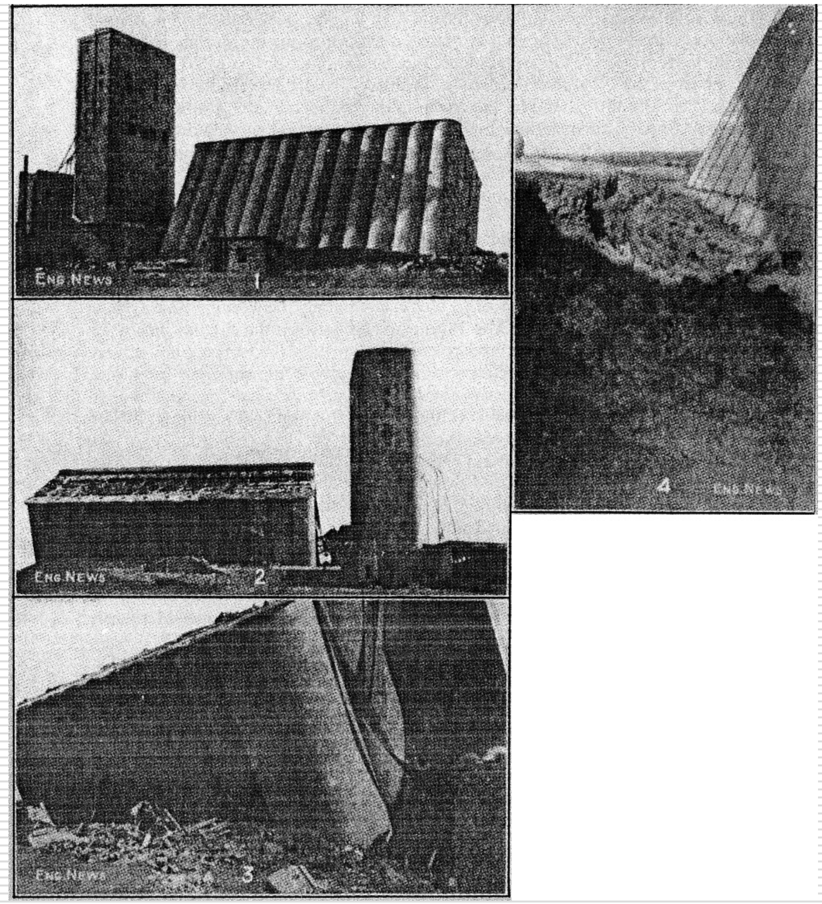
Virtually every structure is supported by *soil or rock*. Those that aren't either *fly*, *float*, or *fall over*".

Richard Handy

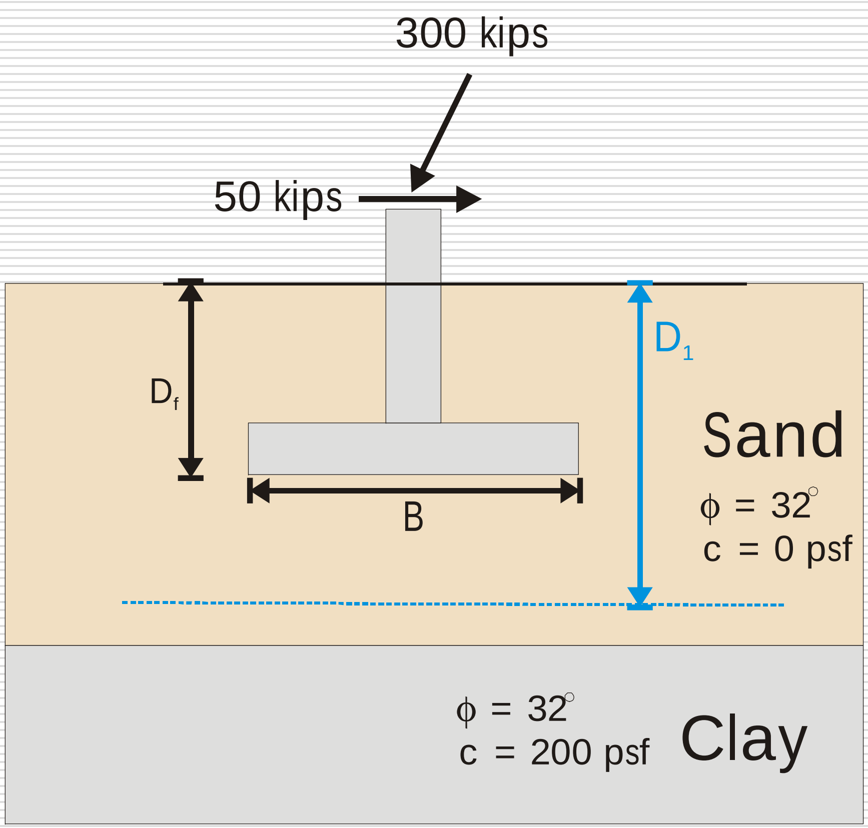
Transcona Grain Elevator Failure







Intuition



What factors affect the bearing capacity and settlement of a footing?

General Concepts

Shallow foundations must satisfy 2 criteria:

- Adequate safety against shear failure of soils
- Do not have excessive settlement

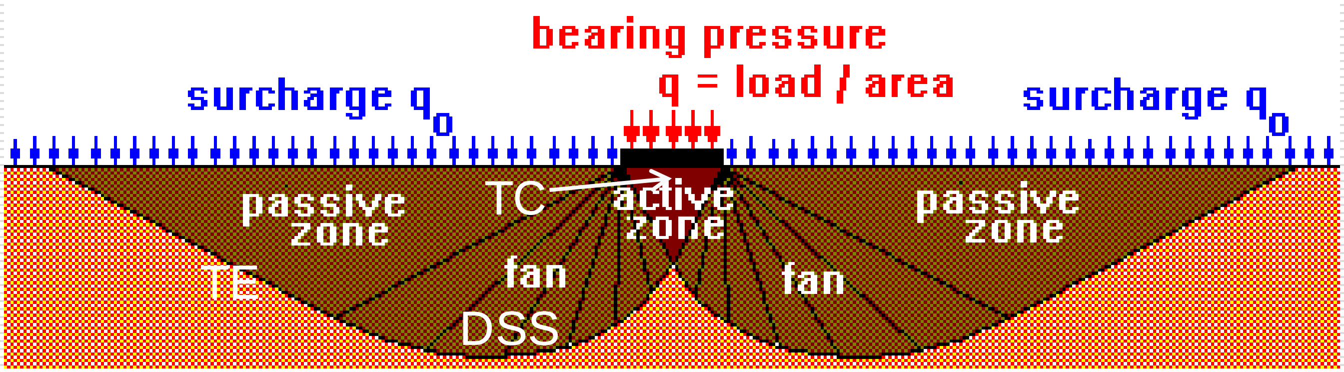
Ultimate Bearing Capacity

- The load per unit area at which there is a shear failure of the soils supporting the foundation

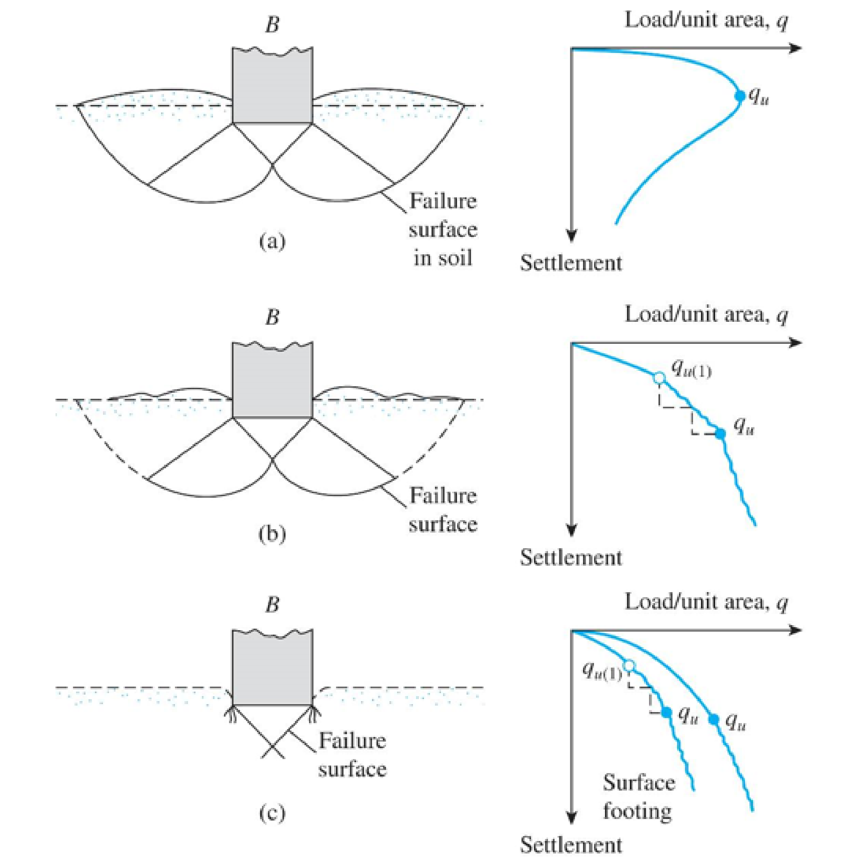
Failure Modes:

- General Shear Failure – sudden failure of soil
- Local Shear Failure – foundation movement by sudden jerks requiring substantial movement for failure to reach ground surface
- Punching Shear Failure – shear failure surface will not reach ground surface
- Modes dependent on soil conditions

BC Zones

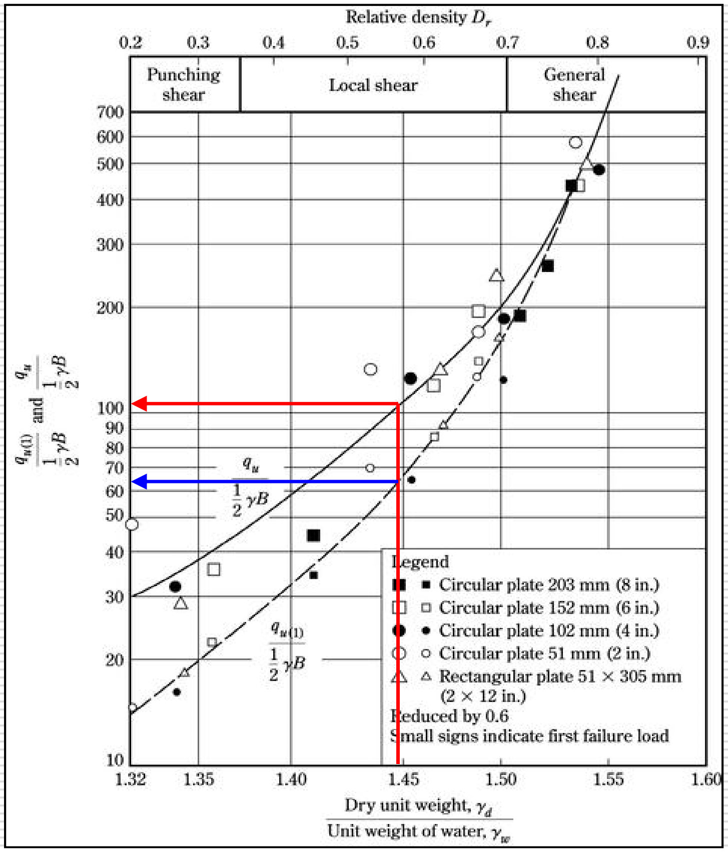

bearing pressure
q = load / area
surcharge q_o
surcharge q_o
TC
active zone
passive zone
passive zone
fan
fan
TE
DSS

Nature of bearing capacity failure in soil



(a) general shear failure;
(b) local shear failure;
(c) punching shear failure
(redrawn after Vesic, 1973)

Variation of $q_{u(1)}/0.5\lambda B$ and $q_u/0.5\lambda B$ for circular and rectangular plates on the surface of a sand

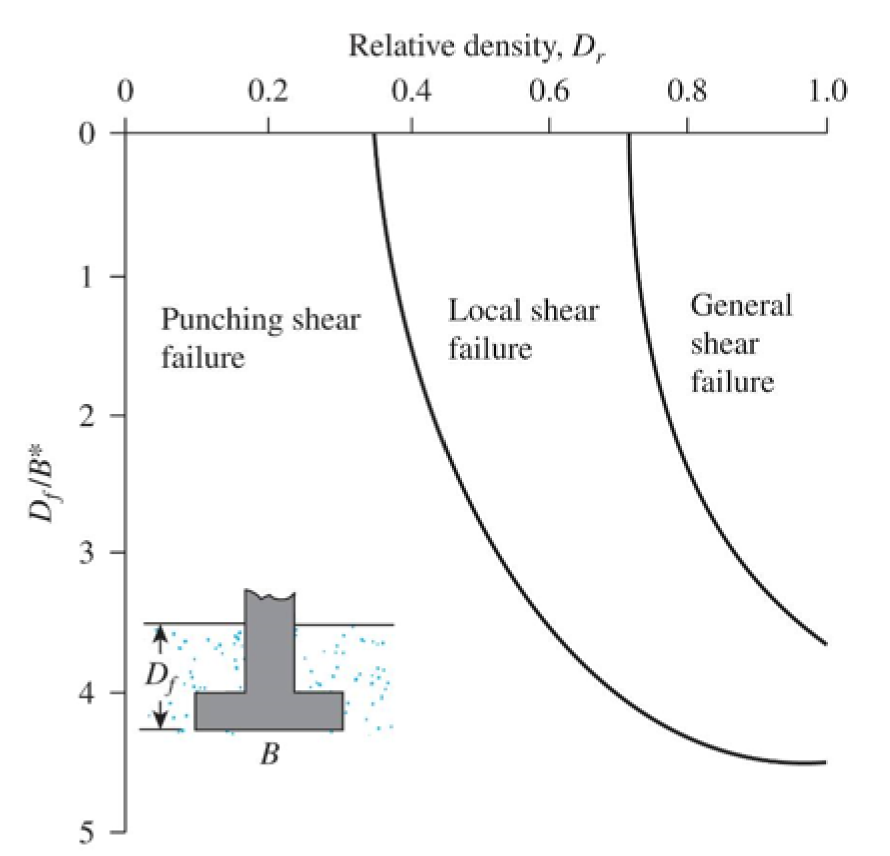


Example

Dry unit weight = 90 pcf
Footing width = 4 feet
$\gamma d/\gamma w$ = 90/62.4 = 1.44
$^1/_2\gamma B$ = 0.5*90*4 = 180 psf
Plot Red Line
$q_u/(^1/_2\gamma B)$ = 105
q_u = 105*180 = 18,900 psf
$q_u(1)$ = 65*180 = 11,700 psf

$q_u(1)$ – circular footing
q_u – rectangular footing

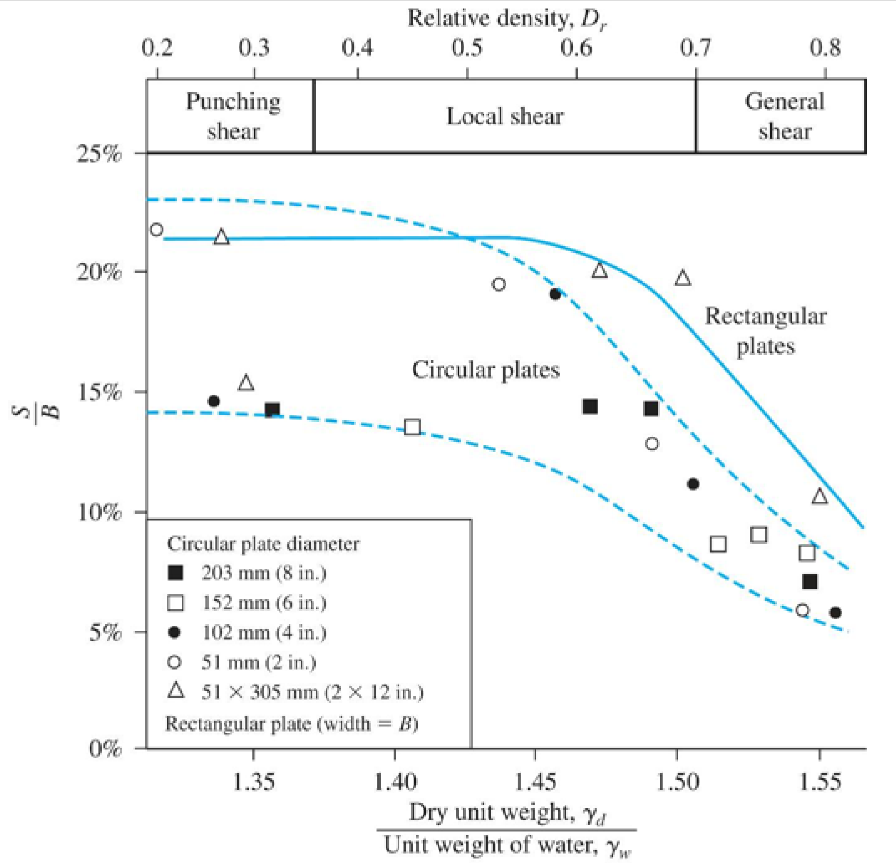
Modes of foundation failure in sand



General shear failure is most likely failure Mode unless:

- Small width footing
- Very loose soils
- Deep embedment

Settlement of circular and rectangular plates at ultimate load (D_f/B = 0) in sand



This nomograph shows that it takes a lot of settlement before full ultimate bearing capacity is achieved.

Settlement Usually Governs

Allowable settlements for structures

- **Strip foundation (masonry) < ¾ inch**
- **Square footing < 1 inch**

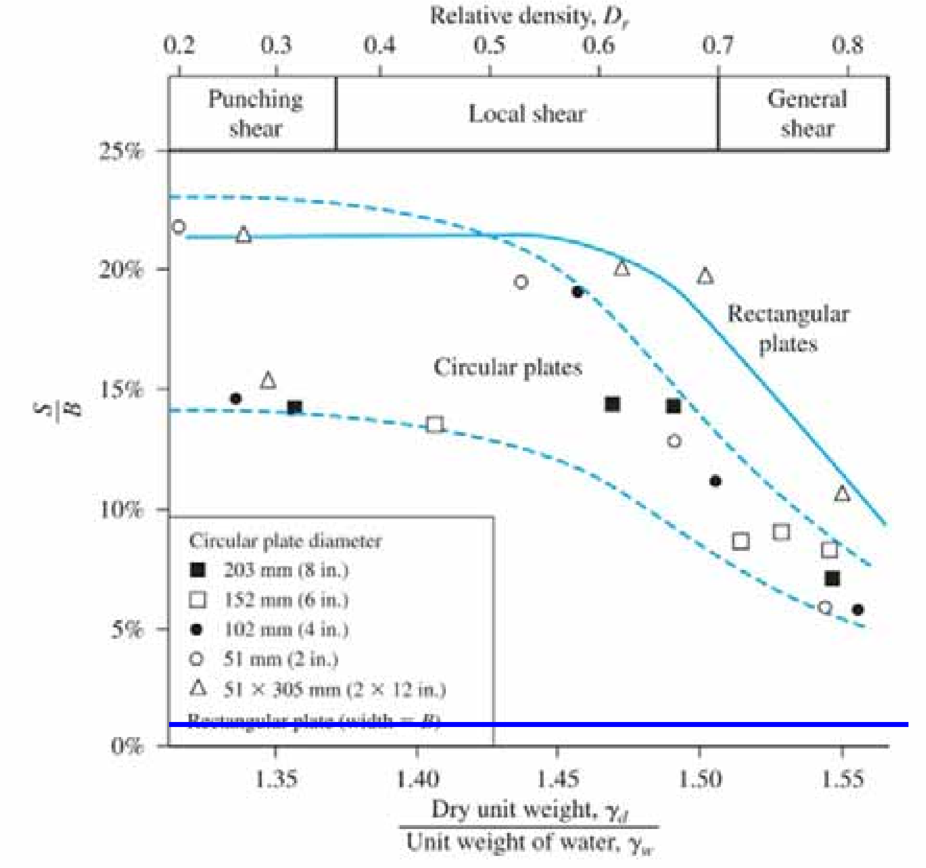
<u>Example</u>

A 150 kip column load is supported by a square footing with an allowable bearing pressure of 2 kips per square foot. Footing with is

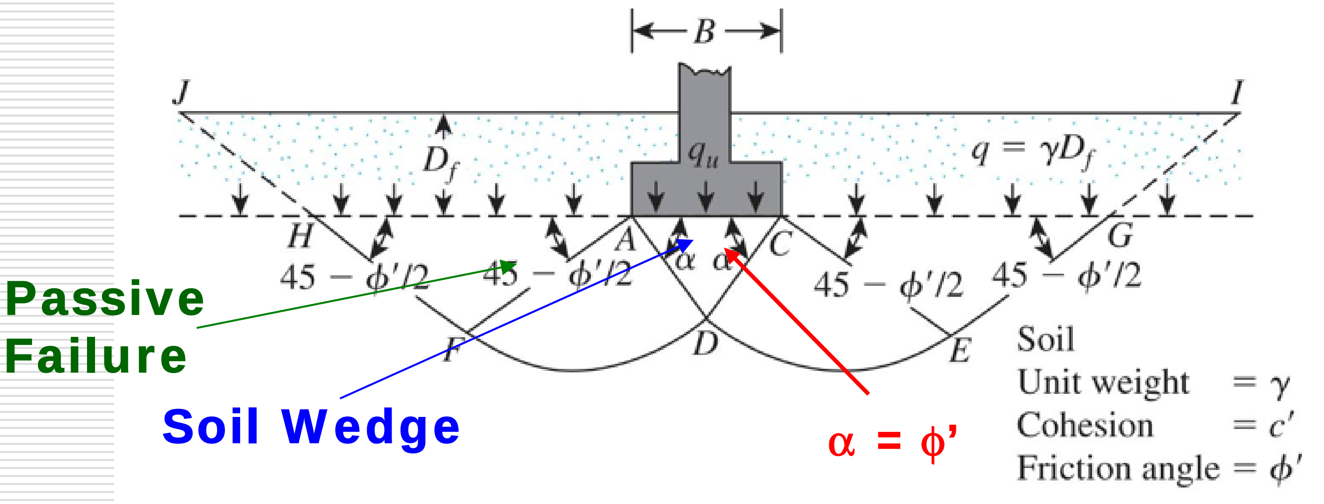
$$\text{Footing Width (B)} = \sqrt{\frac{150}{2}} = 8.7 \text{ feet}$$

For 1 inch settlement S/B = (1/12)/8.7 = 0.0096 or 0.96% about 1%

Example Settlement



Terzaghi Bearing Capacity

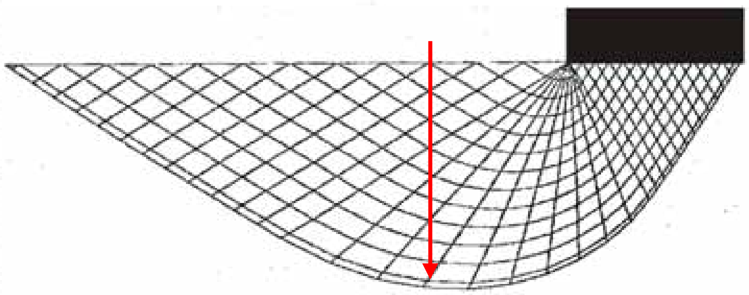


Assumptions

- General shear failure
- Continuous strip foundation (Width/Length approaches zero)
- No contribution by shear J-H and I-G
- Soil above base acts as surcharge load $q = \gamma \bullet Df$

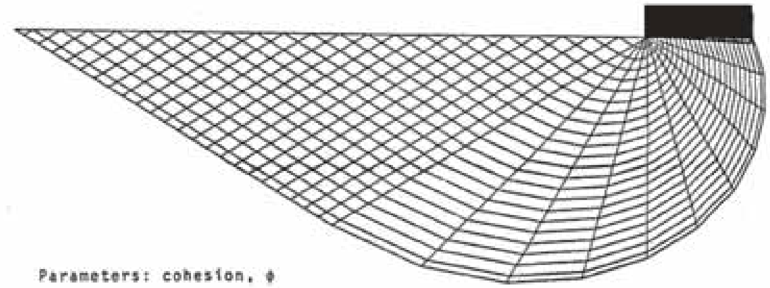
Effects of Cohesion

Typically B to 1.5 B



Parameters: ϕ

Figure 4.12- Plot of General Shear Failure Pattern Generated by Sokolovski's Computer Algorithm (Appendix A).



Parameters: cohesion, ϕ

Figure 4.11- Plot of General Shear Failure Pattern Generated by Sokolovski's Computer Algorithm (Appendix A).

Terzaghi Equation for BC

$$q_u := \frac{\gamma \cdot B \cdot N_\gamma}{2} + C \cdot N_c + \gamma \cdot Df \cdot N_q$$

where:

q_u = ultimate bearing capacity

γ = moist unit weight of soil

B = footing width

C = effective cohesion

D_f = depth of embedment

N_γ, N_c, N_q are bearing capacity factors depending on ϕ

Table 3.1
Or Eq. 3.4 – 3.6

Equation for a continuous strip footing

Modification for Other Shapes

Square Footing

$$q_u := 0.4\gamma \cdot B \cdot N_\gamma + 1.3C \cdot N_c + \gamma \cdot Df \cdot N_q$$

Circular Footing

$$q_u := 0.3\gamma \cdot B \cdot N_\gamma + 1.3C \cdot N_c + \gamma \cdot Df \cdot N_q$$

Local Shear Equations

$$q_u := \frac{\gamma \cdot B \cdot N'_\gamma}{2} + \frac{2}{3} \cdot C \cdot N'_c + \gamma \cdot Df \cdot N'_q$$

strip

$$q_u := 0.4\gamma \cdot B \cdot N'_\gamma + 0.867 C \cdot N'_c + \gamma \cdot Df \cdot N'_q$$

square

$$q_u := 0.3 \cdot \gamma \cdot B \cdot N'_\gamma + 0.867 C \cdot N'_c + \gamma \cdot Df \cdot N'_q$$

circular

Where N'γ, N'c, and N'q are BC factors Nγ, Nc, and Nq modified by substituting ϕ' with

$$\overline{\phi'} := \tan^{-1}\left(\frac{2}{3} \cdot \tan(\phi')\right)$$

Table 3.2

Factor of Safety

$$q_{allowable} = q_{ultimate} / FS$$

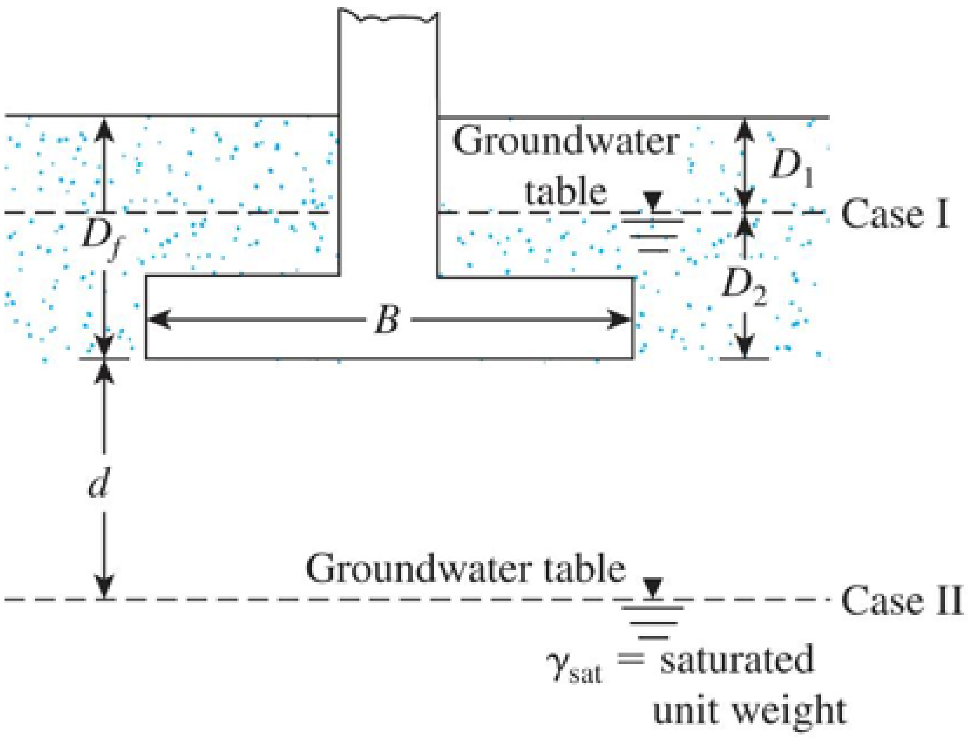
Selected factor of safety is dependent on:

- Amount of subsurface information available
- How accurate the subsurface data
- Amount of structural information available
- Sustained load and live load development
- Potential for future changes at the site (flooding, erosion)
- Experience of the engineer in those soil conditions

The greater the confidence in the available information the lower the safety factor can be.

- Minimum safety factor of 2 for good accurate data
- Safety factor of 4 for very low level of confidence

Modification of bearing capacity equations for water table



Case I

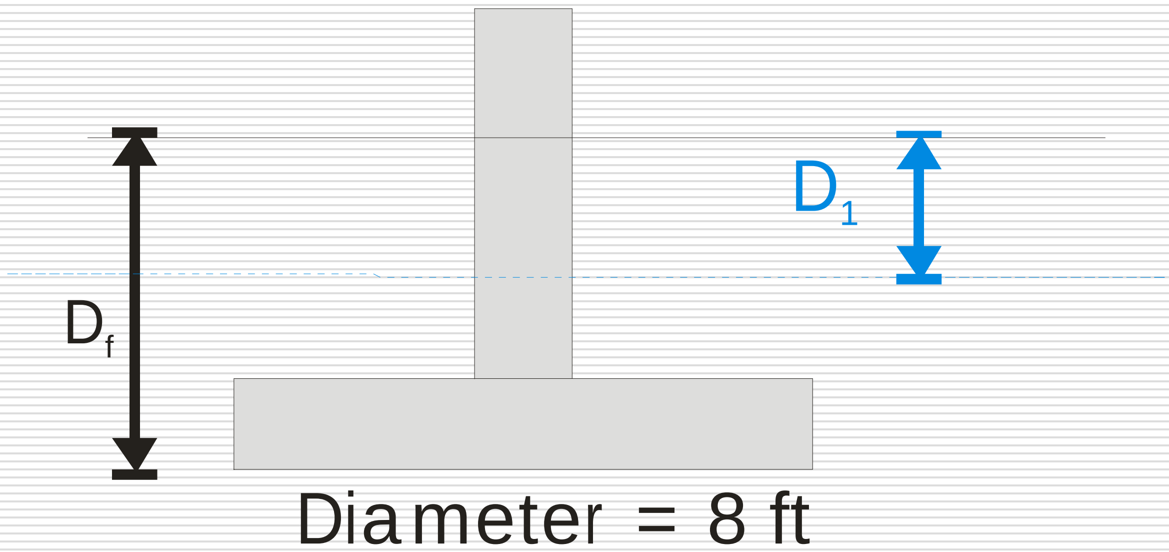
$$\gamma \cdot D_f = \gamma \cdot D1 + (\gamma_{sat} - \gamma_w) \cdot D2$$

Case II

$$\gamma' = \gamma_{sat} - \gamma_w$$

$$\bar{\gamma} = \gamma' + \frac{d}{B} \cdot (\gamma - \gamma')$$

A Circular Foundation



γ = 110 pcf
γ_{sat} = 120 pcf
ϕ = 33
C = 200 psf

D_f = 4 ft
D_1 = 2 ft

Solution

First, we have a circular foundation, so we use $q_u = 0.3\gamma B N_\gamma + 1.3 C N_c + \gamma D_f N_q$

Determine $\gamma D_f = q = \gamma D_1 + (\gamma_{sat} - \gamma_w) D_2 = (110)(2) + (120-62.4)(4-2) = 335.2$ psf

Next, determine γ for first part of equation $\gamma = \gamma' = \gamma_{sat} - \gamma_w = 120-62.4 = 57.6$ pcf

Obtain N_γ, N_c and N_q for $\phi = 33$. From Table 3.1 (page 139) $N_\gamma = 31.94$, $N_c = 48.09$, $N_q = 32.23$

$q_u = 0.3(57.6)(8)(31.94) + 1.3(200)(48.09) + 335.2(32.23) = 27722.3$ psf

Structural Information

Structural information is critical to your analysis of foundations.

- **The range of loads varies the sizes of foundations.**
- **The sizes of foundations is a factor in bearing capacity.**
- **The sizes of foundations effects the range of settlements.**
- **Very high loads could require excessive footing sizes and a change in foundation type.**
- **A high variation in adjacent loads could lead to excessive differential settlements.**

Typically, the project structural engineer will provide loading conditions for the project. Your experience in local soil conditions gives you a starting point for bearing capacity analyses. You are never given footing sizes during design. You may have footing sizes after design to analyze changed conditions.

Other Conditions

Not All Conditions Are Simple

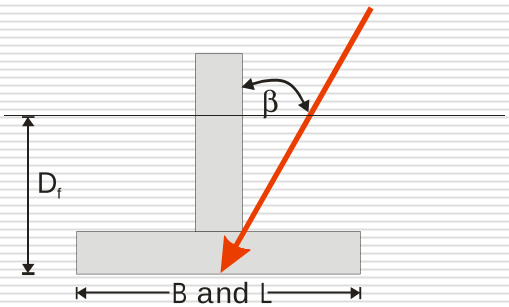
Karl Terzaghi's bearing capacity is for
a special case of a strip foundation on flat
ground and embeded at nominal depths.
Previous modifications are based on
foundation shape. There are more complex
scenarios that require further changes.

General Bearing Capacity Equation

$$q_u := \frac{1}{2}\cdot\gamma\cdot B\cdot N_\gamma\cdot F_{\gamma s}\cdot F_{\gamma d}\cdot F_{\gamma i} + c'\cdot N_c\cdot F_{cs}\cdot F_{cd}\cdot F_{ci} + \gamma\cdot D_f\cdot N_q\cdot F_{qs}\cdot F_{qd}\cdot F_{qi}$$

Now considers footing shape, depth, load inclination, and change in bearing capacity factors.

- Bearing capacity factors now based on passive parameters with = 45 + $\phi'/2$.
- Shape factors consider intermediate shapes (rectangles).
- Depth factors consider the effects of embedment depth.
- Inclination factors consider non-vertical loading conditions.



Modified Bearing Capacity Factors

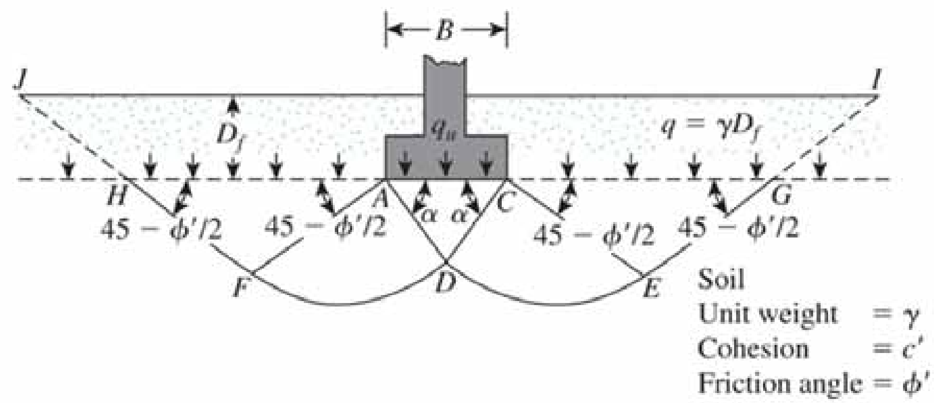
Studies indicate that $\alpha := 45 + \frac{\phi'}{2}$ (passive)

$$N_q := \tan^2\left(45 + \frac{\phi'}{2}\right)\cdot e^{\pi\cdot\tan(\phi')}$$

Table 3.3

$$N_c := \left(N_q - 1\right)\cot\left(\phi'\right)$$

$$N_\gamma := 2\cdot\left(N_q + 1\right)\tan\left(\phi'\right)$$



Shape, Depth & Inclination Factors

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Table 3.4 Shape, Depth and Inclination Factors (DeBeer (1970); Hansen (1970); Meyerhof (1963); Meyerhof and Hanna (1981))

Factor	Relationship	Reference
Shape	$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$	DeBeer (1970)
	$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi'$	
	$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$	
Depth	$\frac{D_f}{B} \leq 1$	Hansen (1970)
	For $\phi = 0$:	
	$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$	
	$F_{qd} = 1$	
	$F_{\gamma d} = 1$	
	For $\phi' > 0$:	
	$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$	
	$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right)$	
	$F_{\gamma d} = 1$	
	$\frac{D_f}{B} > 1$	
	For $\phi = 0$:	
	$F_{cd} = 1 + 0.4\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$	
	$F_{qd} = 1$	
	$F_{\gamma d} = 1$	
	For $\phi' > 0$:	
	$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$	
	$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$	
	$F_{\gamma d} = 1$	
Inclination	$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$	Meyerhof (1963); Hanna and Meyerhof (1981)
	$F_{\gamma i} = \left(1 - \frac{\beta}{\phi'}\right)$	
	β = inclination of the load on the foundation with respect to the vertical	

Effect of Compressibility

Terzaghi's classic equation for bearing capacity was for a general shear failure. It was modified to account for local shear failure when the Engineer felt that failure mode was applicable.

The general bearing capacity equation can take failure mode into account by additional factors multiplying the 3 portions of the formula as suggested by Vesic.

The rigidity index is used to take compressibility into account and modify the results. It answers the question – "Which failure mode do I have?"

Rigidity Index

$$I_r := \frac{G_s}{c' + q' \cdot \tan(\phi')}$$

where

Gs = shear modulus of soils $G_s := \frac{E_s}{2 \cdot (1 + \mu_s)}$

q' = effective overburden pressure

Both for soils at a depth of D_f + B / 2

Critical Rigidity Index

When does soil compressibility become an issue? Use critical rigidity index.

$$I_{r(cr)} := 0.5 \cdot \left[e^{\left[\left(3.3 - 0.45 \cdot \frac{B}{L} \right) \cdot \cot\left(45 - \frac{\phi'}{2}\right) \right]} \right]$$

If $I_r > I_{r(cr)}$ then $F_{cc} = F_{qc} = F_{\gamma c} = 1$

Table 3.6 for I_r on page 154

Table 3.6 Variation of $I_{r(cr)}$ with ϕ' and B/L

ϕ' (deg)	$I_{r(cr)}$					
	$B/L = 0$	$B/L = 0.2$	$B/L = 0.4$	$B/L = 0.6$	$B/L = 0.8$	$B/L = 1.0$
0	13.56	12.39	11.32	10.35	9.46	8.64
5	18.30	16.59	15.04	13.63	12.36	11.20
10	25.53	22.93	20.60	18.50	16.62	14.93
15	36.85	32.77	29.14	25.92	23.05	20.49
20	55.66	48.95	43.04	37.85	33.29	29.27
25	88.93	77.21	67.04	58.20	50.53	43.88
30	151.78	129.88	111.13	95.09	81.36	69.62
35	283.20	238.24	200.41	168.59	141.82	119.31
40	593.09	488.97	403.13	332.35	274.01	225.90
45	1440.94	1159.56	933.19	750.90	604.26	486.26

Compressibility Factors

If $I_r < I_{r(cr)}$

$$F_{\gamma c} = F_{qc} = \exp\{(-4.4+0.6\frac{B}{L})\cdot\tan\phi' + [\frac{(3.07\sin\phi')(\log 2I_r)}{1+\sin\phi'}]\}$$

When $\phi' = 0$, then $F_{\gamma c} = F_{qc} = 1$

When $\phi' = 0$

$$F_{cc} = 0.32 + 0.12\frac{B}{L} + 0.60\log I_r$$

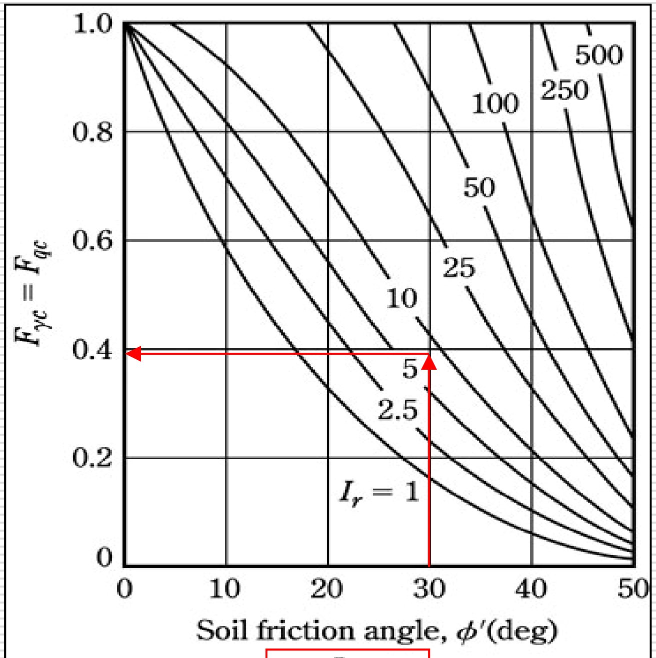
When $\phi' > 0$

$$F_{cc} := F_{qc} - \frac{1 - F_{qc}}{N_q \cdot \tan(\phi')}$$

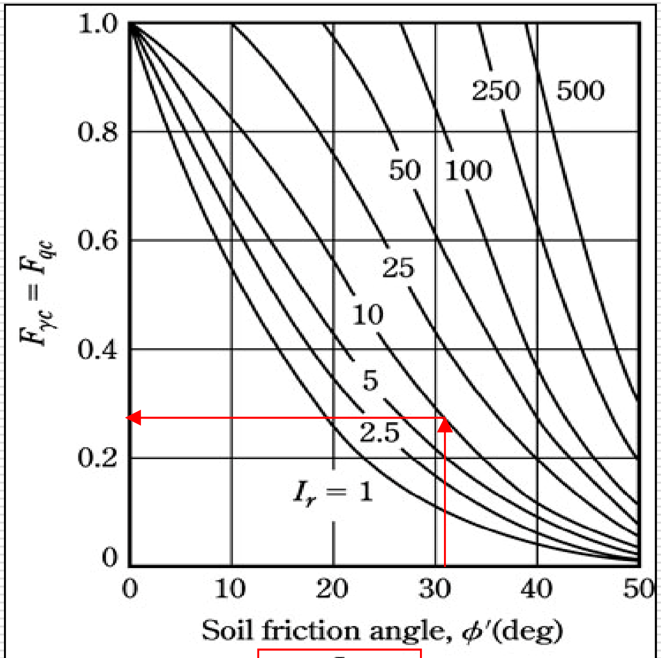
Modified General Bearing Capacity Equation

$$q_u := \frac{1}{2}\cdot\gamma\cdot B\cdot N_\gamma\cdot F_{\gamma s}\cdot F_{\gamma d}\cdot F_{\gamma i}\cdot F_{\gamma c} + c'\cdot N_c\cdot F_{cs}\cdot F_{cd}\cdot F_{ci}\cdot F_{cc} + \gamma\cdot D_f\cdot N_q\cdot F_{qs}\cdot F_{qd}\cdot F_{qi}\cdot F_{qc}$$

Variation of $F_{\gamma c} = F_{qc}$ with I_r and ϕ'



(a) $\frac{L}{B} = 1$



(b) $\frac{L}{B} > 5$

No Footing Width?

- Assume $q_{allowable}$
- Calculate B
- Insert B into BC equation
- Calculate q_{ult}
- Determine FS
- Reinterate as needed.

Eccentricity

All previous shallow footing problems had one thing in common:

A vertical load has been applied at the center of the foundation.

Eccentricity occurs when loading conditions shift the center of loading. This occurs when:

- **A moment is applied**
- **The load is shifted off center by design**
- **Figure 3.14 p. 158**

Eccentric Failure

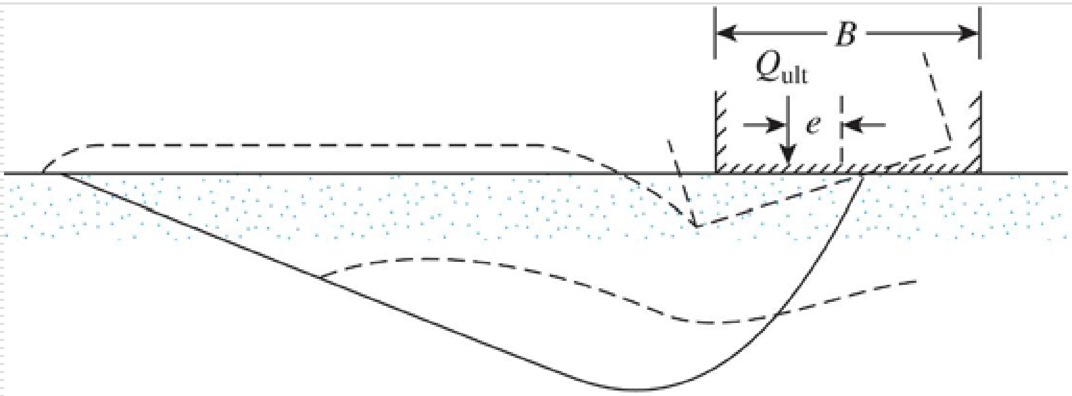
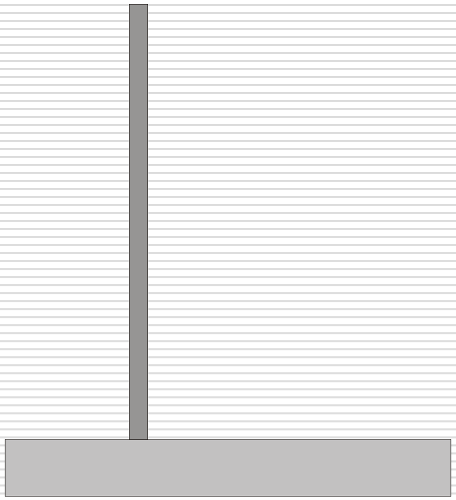


Figure 3.14 Nature of failure surface in soil supporting a strip foundation subjected to eccentric loading (*Note:* $D_f = 0$; Q_{ult} is ultimate load per unit length of foundation)

Eccentricity Examples

Signs create moments by wind loading

Retaining walls or structural conditions



Eccentrically loaded foundations

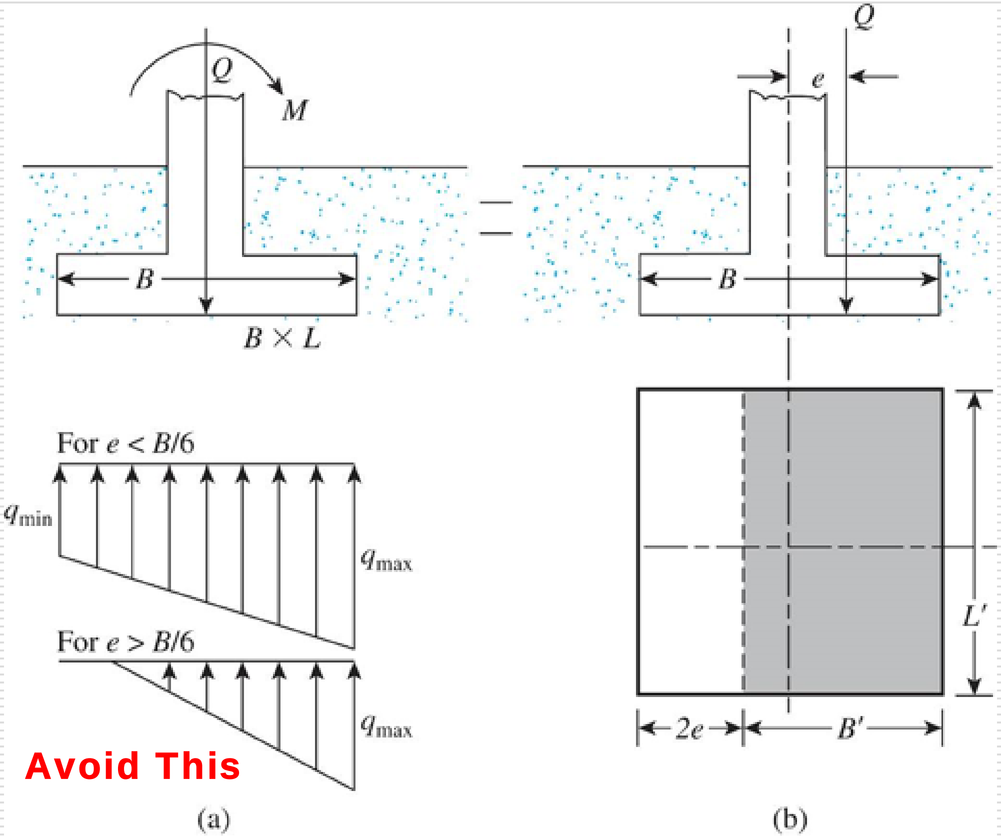
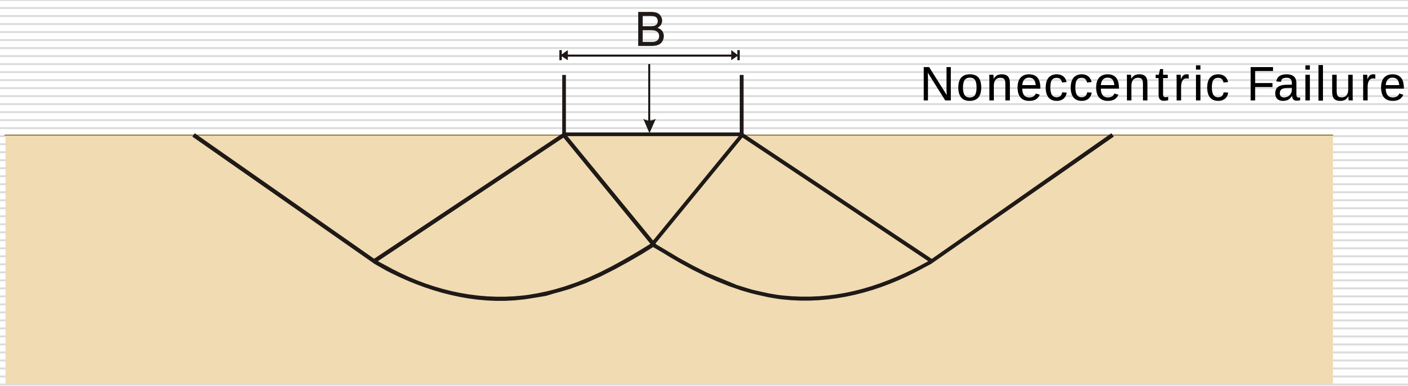
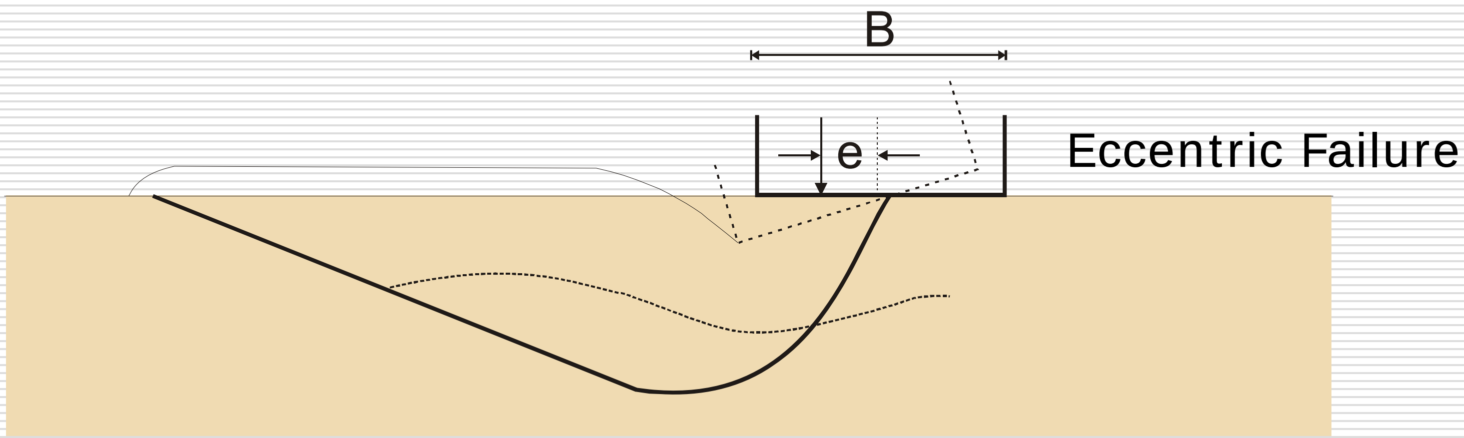


Figure 3.13 Eccentrically loaded foundations

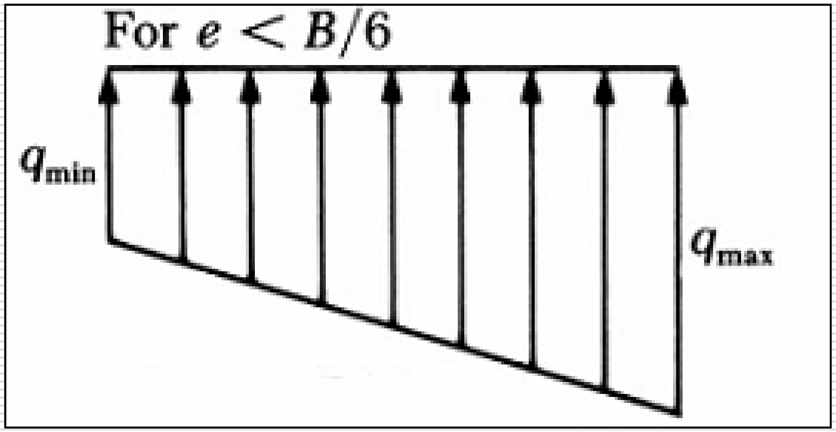
Change Mode of Failure





Max & Min Pressures

$$q_{min} := \frac{Q}{B \cdot L} - \frac{6\,M}{B^2 \cdot L}$$ ← Moments → $$q_{max} := \frac{Q}{B \cdot L} + \frac{6\,M}{B^2 \cdot L}$$

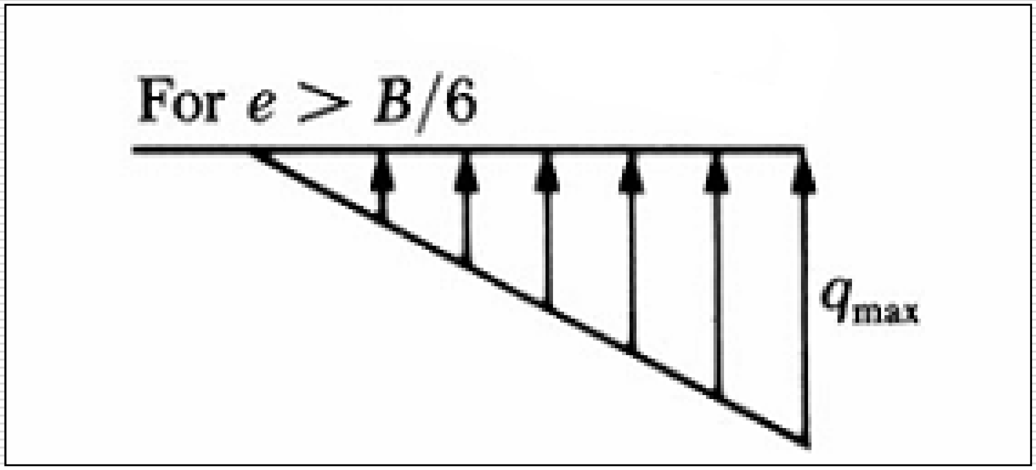


$$e = \frac{M}{Q}$$

$$q_{min} := \frac{Q}{B \cdot L} \cdot \left(1 - \frac{6\,e}{B}\right)$$ ← Eccentricity → $$q_{max} := \frac{Q}{B \cdot L} \cdot \left(1 + \frac{6\,e}{B}\right)$$

Q = vertical load, M = moment, e = eccentricity

Excess Eccentricity



$$q_{max} := \frac{4\,Q}{3\,L\cdot(B - 2\,e)} \qquad q_{min} = 0$$

Effective Width

Modify the footing width "B" for the effects of eccentricity

B' = effective width = B – 2e
L' = effective length = L

Use B' and L' substituting for B and L in general bearing capacity equation

For F_{cd}, F_{qd} & $F_{\gamma d}$ do not replace B with B'

$Q_{ult} = q_u(B')(L')$

Prakash and Saran

Vertically & Eccentrically Loaded Continuous (Strip) Footing

$$Qult := B\cdot\left(c\cdot N_{c(e)} + q\cdot N_{q(e)} + 0.5\gamma\cdot B\cdot N_{\gamma(e)}\right)$$

Vertically & Eccentrically Loaded Rectangular Footing

$$Qult := B\cdot L\cdot\left(c\cdot N_{c(e)}\cdot F_{cs(e)} + q\cdot N_{q(e)}\cdot F_{qs(e)} + 0.5\gamma\cdot B\cdot N_{\gamma(e)}\cdot F_{\gamma s(e)}\right)$$

Shape Factors

$$F_{cs}(e) := 1.2 - 0.025 \cdot \frac{L}{B} \quad <= 1$$

$$F_{qs}(e) := 1$$

$$F_{\gamma s}(e) := 1.0 + \left[\left(2 \cdot \frac{e}{B}\right) - 0.68\right] \cdot \left(\frac{B}{L}\right) + \left[0.43 - \left(\frac{3}{2}\right) \cdot \left(\frac{e}{B}\right)\right] \cdot \left(\frac{B}{L}\right)^2$$

$N_{c(e)}$

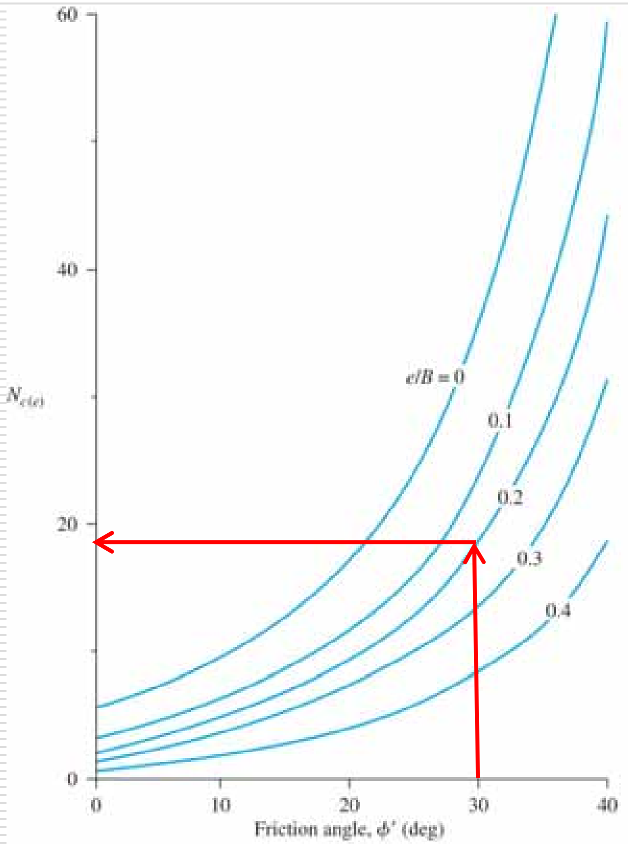
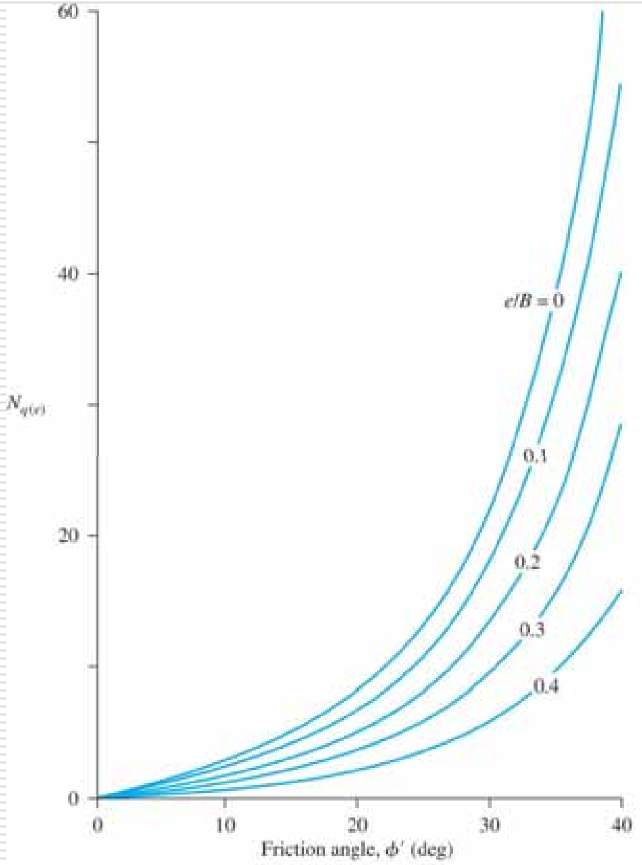


Figure 3.15 Variation of $N_{c(e)}$ with ϕ'

$\phi' = 40°$

e/B	$N_{c(e)}$
0	94.83
0.1	66.60
0.2	54.45
0.3	36.3
0.4	18.15

$N_{q(e)}$

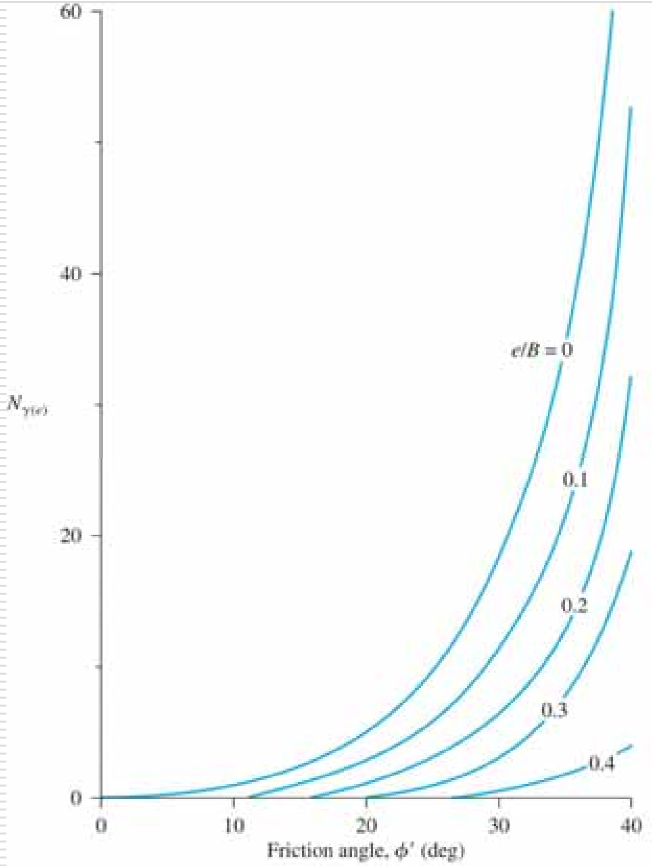


$\phi' = 40°$

e/B	$N_{q(e)}$
0	81.27
0.1	56.09
0.2	45.18
0.3	30.18
0.4	15.06

Figure 3.16 Variation of $N_{q(e)}$ with ϕ'

$N_{\gamma(e)}$



$\phi' = 40°$

e/B	$N_{\gamma(e)}$
0	115.80
0.1	71.80
0.2	41.60
0.3	18.50
0.4	4.62

Figure 3.17 Variation of $N_{\gamma(e)}$ with ϕ'

Two Way Eccentricity

Eccentricity can be one way and two way.

Examples of two way eccentricity:

- **Sign foundation**
- **Water tower with tank on top**

Now we have moments that effect both footing dimensions B and L.

Analysis of foundation with two-way eccentricity

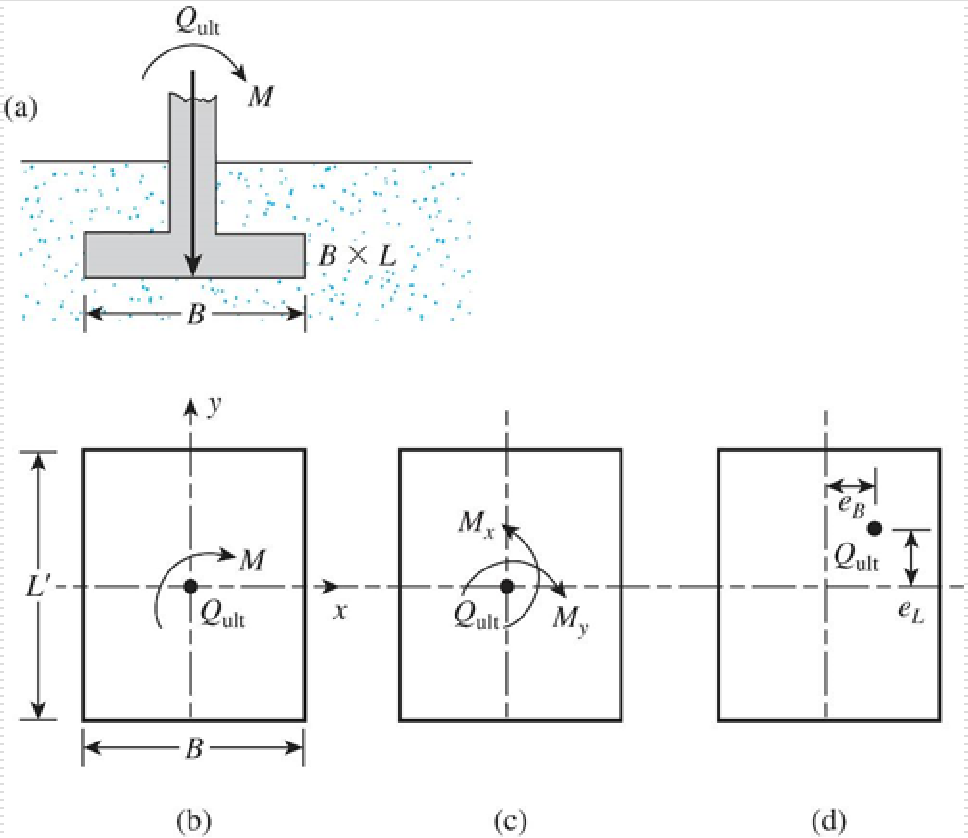
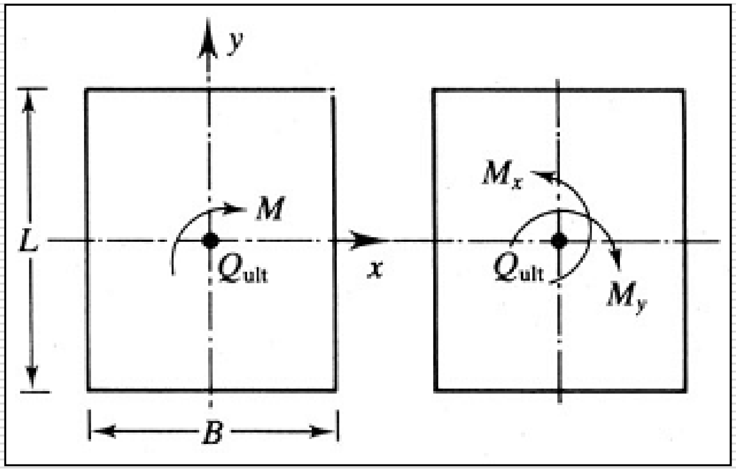


Figure 3.19 Analysis of foundation with two-way eccentricity

1 WAY VERSUS 2 WAY

One way moment



Two way moment

$$e_L := \frac{M_x}{Q_{ult}} \qquad e_B := \frac{M_y}{Q_{ult}}$$

$$Q_{ult} = q'_u \bullet A' \qquad A' = B' \bullet L'$$

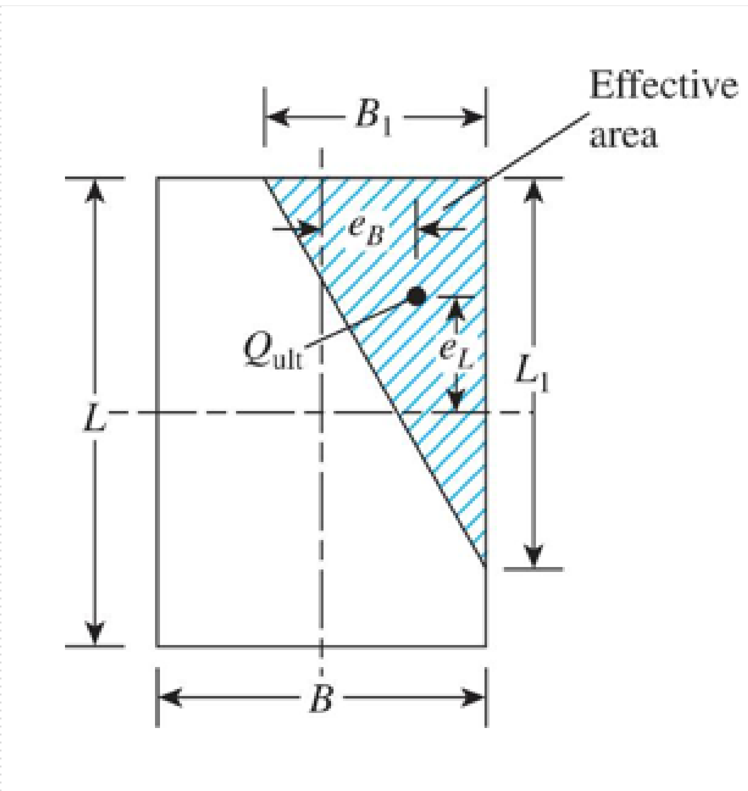
where q'_u is calculated from bearing capacity equation

Two Way Eccentricity Cases

Depending on loading conditions two way eccentricity is analyzed one of five ways.

1. **$e_L/L >= 1/6$ and $e_B/B >= 1/6$**
2. **$e_L/L < 1/2$ and $e_B/B < 1/6$**
3. **$e_L/L < 1/6$ and $e_B/B < 1/2$**
4. **$e_L/L < 1/6$ and $e_B/B < 1/6$**
5. **Circular footing – always 1 way**

Effective area for the case of $e_L/L >= 1/6$ and $e_B/B >= 1/6$



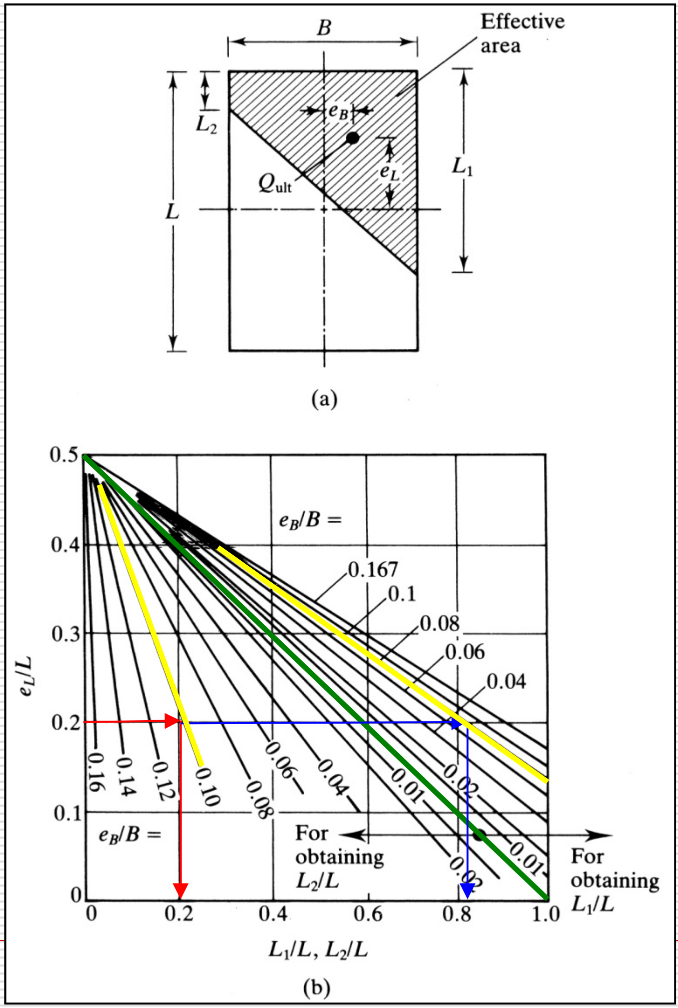
$$B_1 := B\cdot\left(1.5 - \frac{3\,e_B}{B}\right)$$

$$L_1 := L\cdot\left(1.5 - \frac{3\,e_L}{B}\right)$$

$$A' = \tfrac{1}{2} B_1 \bullet L_1$$

L' = larger of B_1 or L_1
SO B' = A'/L'

Effective area for the case of $e_L/L < 0.5$ and $0 < e_B/B < 1/6$

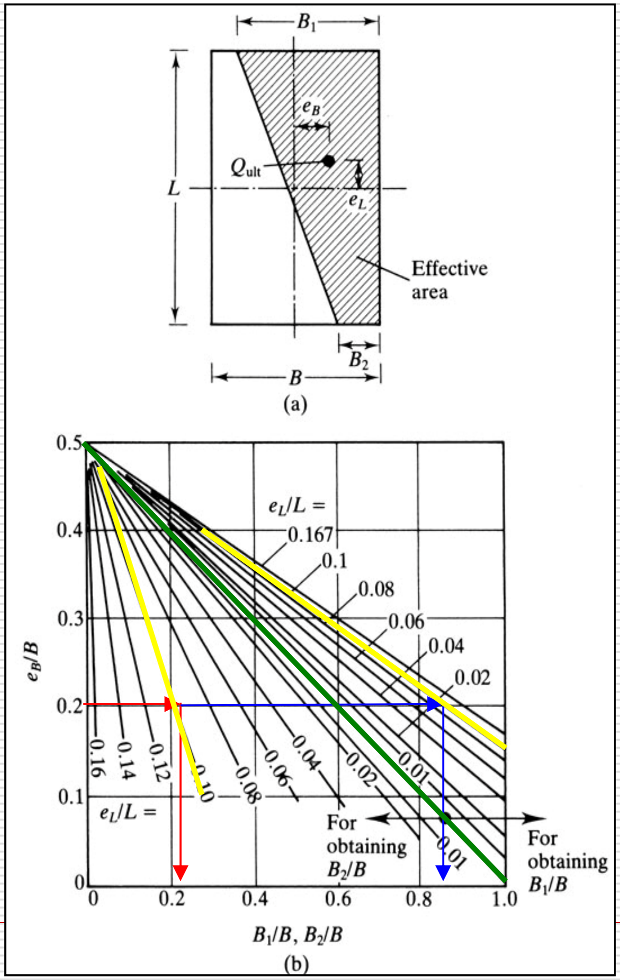


$A' = \frac{1}{2}(L_1 + L_2)B$

L' = larger of L_1 or L_2

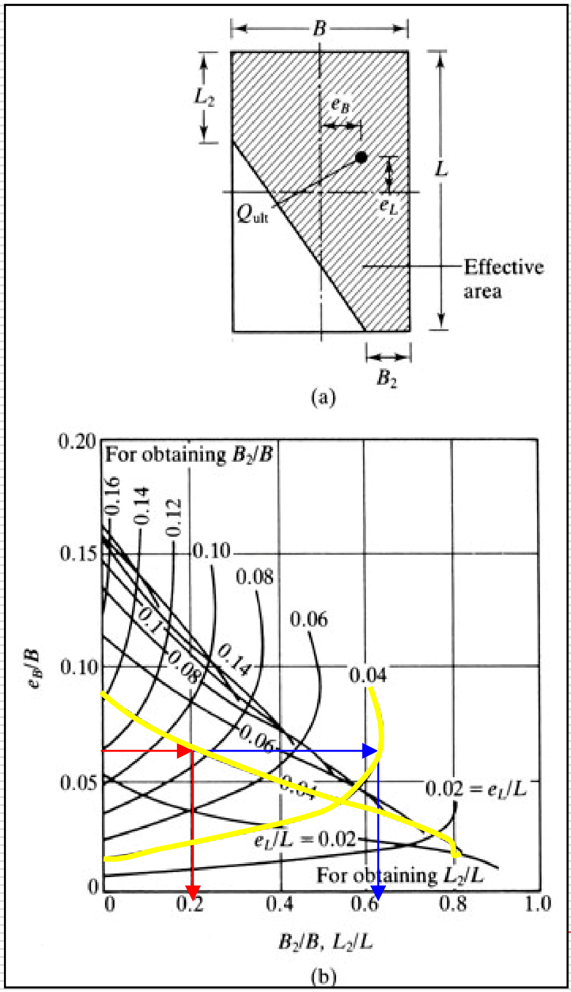
$B' = A' / L'$

Effective area for the case of $e_L/L < 1/6$ and $0 < eB/B < 0.5$



$A' = ½(B_1 + B_2)L$
$B' = A'/L$
$L' = L$

Effective area for the case of $e_L/L < 1/6$ and $e_B/B < 1/6$



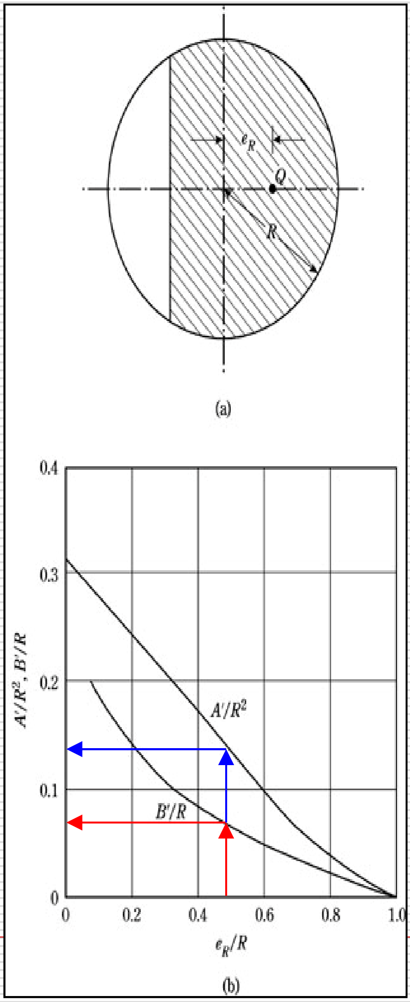
$A' = L_2B + \frac{1}{2}(B + B_2)(L - L_2)$

$B' = A' / L$

$L' = L$

2 overlap

Effective area for circular foundation



Eccentricity on a circular footing is always one way.

Use Table 3.8 to obtain A' and B' and solve for L' = A' / B'

Homework

From Chapter 3

CE 430

- 3.1a & c
- 3.2
- 3.3 a & c
- 3.8

CE 530

Same as CE 430 plus

- 3.9
- 3.13