Principles of Foundation Engineering

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Chapter 4

Ultimate Bearing Capacity Of Shallow Foundations: Special Cases

Special Cases

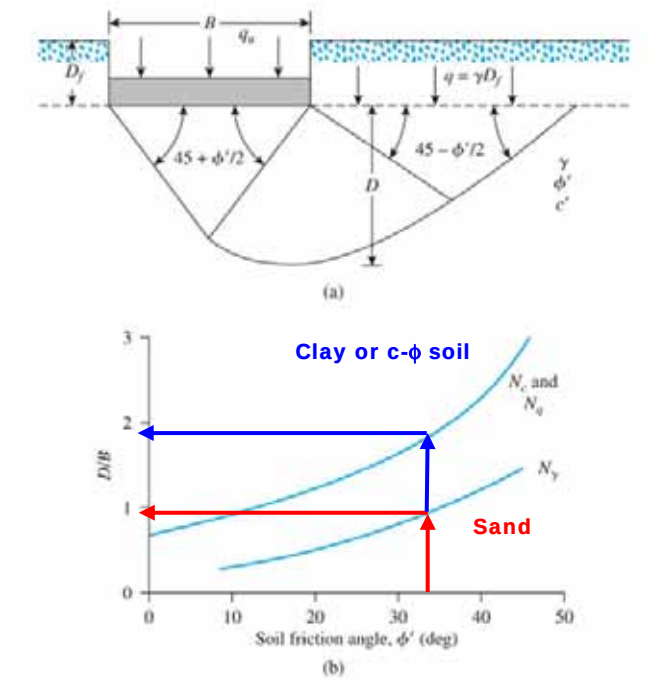
All our analyses to this point have assumed the following:

- **The soil supporting the foundation below its base is homogeneous and extends to great depth.**
- **The ground surface is horizontal.**

This is not always the case.

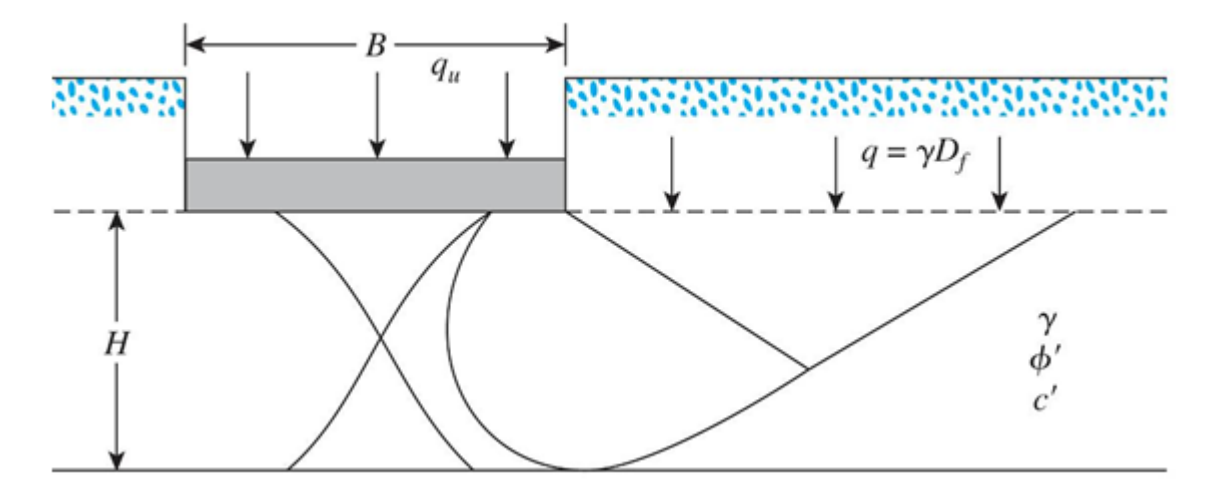
- **There will be cases where more than one soil type will be present within the failure zone. Such cases include:**
  1. **Rigid base that prevents full development of general shear**
  2. **Stronger soil underlain by weaker soil**
- **The foundation will be on or near a sloped ground surface.**

Non-Rigid Base



(a) Failure surface under a rough continuous foundation;
(b) variation of *D*/*B* with soil friction angle ϕ'

Rigid Base Failure Mode



H<=B

Failure surface under a rough, continuous foundation with a rigid, rough base located at a shallow depth (ie. Rock)

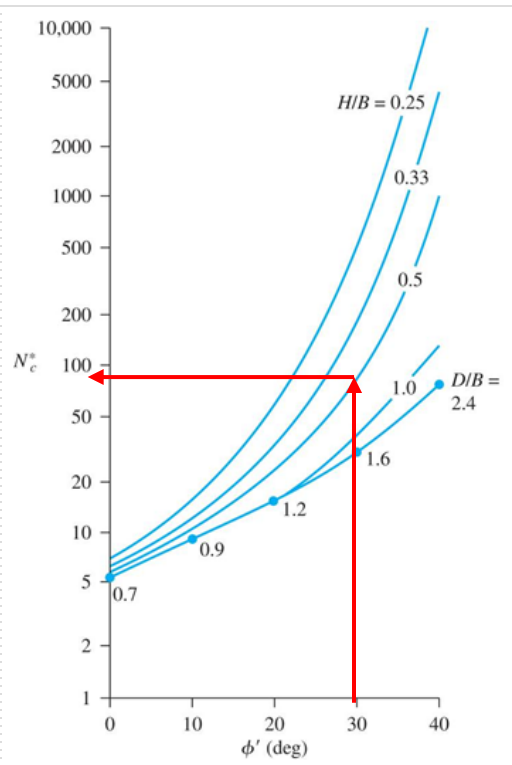
Rigid Base

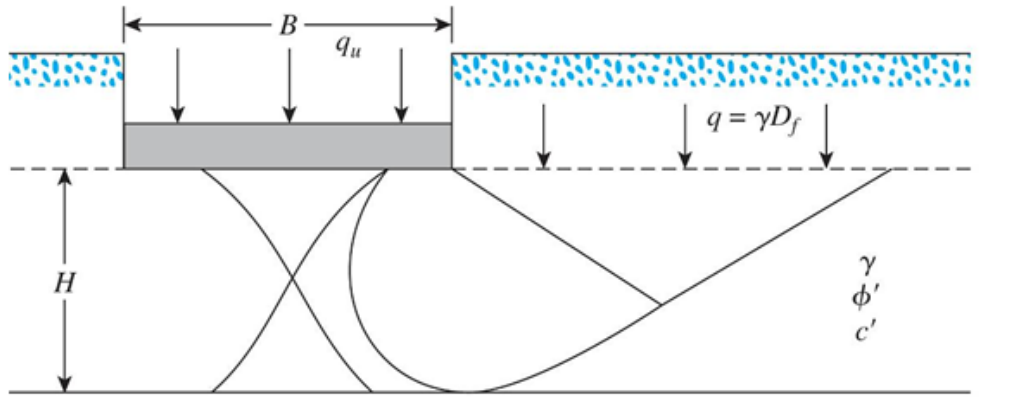
$$q_u := \frac{\gamma \cdot B \cdot N_\gamma^*}{2} + c' \cdot nN_c^* + \gamma \cdot D_f \cdot N_q^*$$

Same equation we have been using except we modify the bearing capacity factors based on the depth to the rigid base and its ratio with footing width.

Note: we do not take the strength of the rigid base into account in our analyses. It is assumed to be strong enough not to fail.

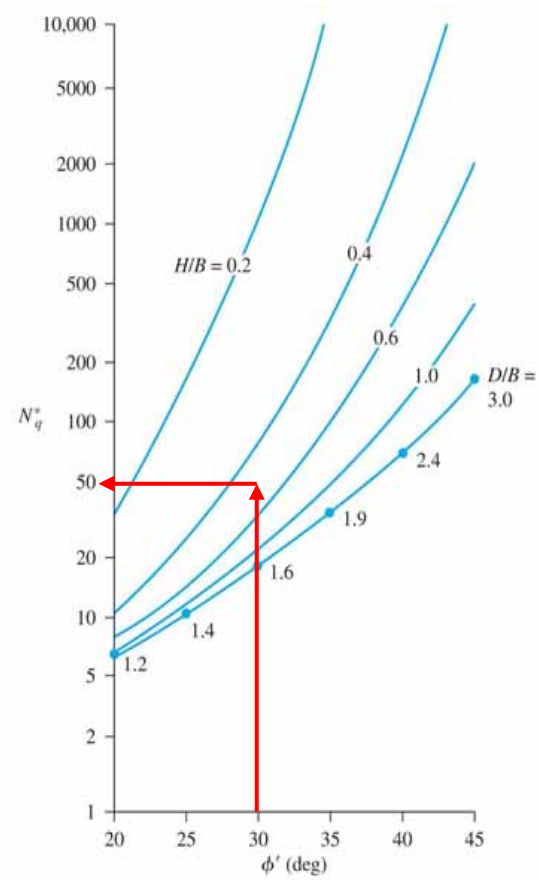
Mandel and Salencon's bearing capacity factor N^*_c (Equation 4.2)

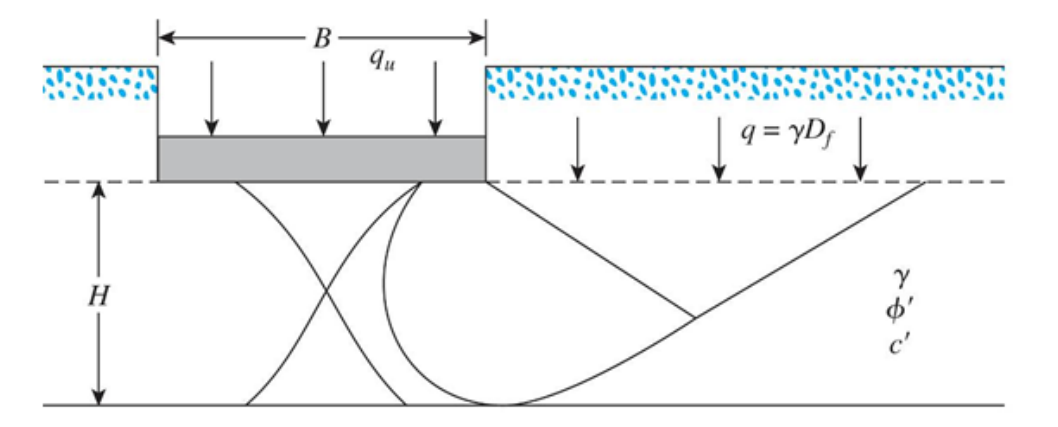




Nc = 37
Nc* = 80

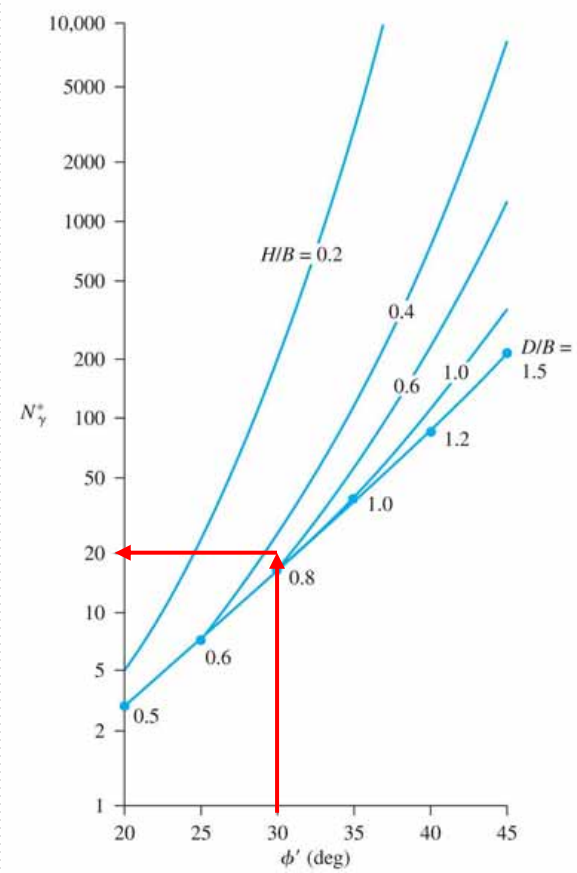
Mandel and Salencon's bearing capacity factor N^*_q (Equation 4.2)

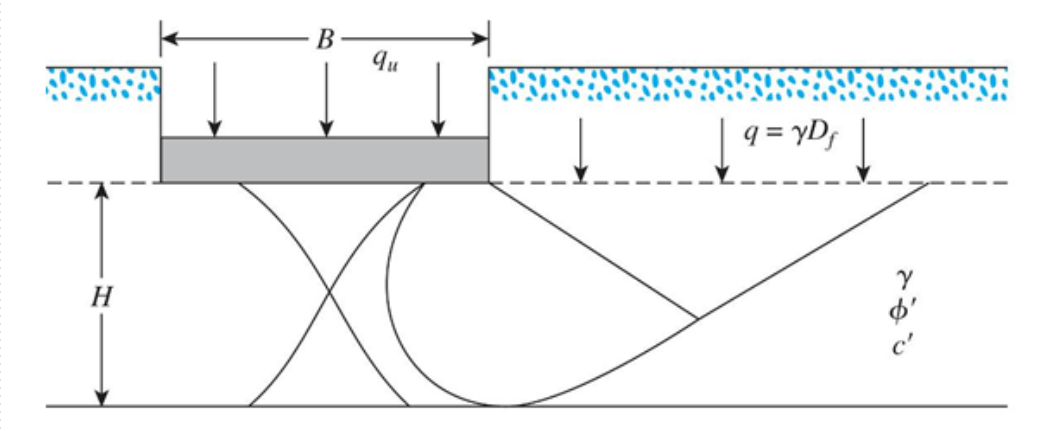




Nq = 22
Nq* = 50

Mandel and Salencon's bearing capacity factor N^*_γ (Equation 4.2)





Nγ = 19
Nγ* = 20

Two Simplified Cases

For sands c=0 (Rectangular or Square) – What about continuous?

$$q_u := \frac{\gamma \cdot B \cdot N_\gamma^* \cdot F_{\gamma s}^*}{2} + \gamma \cdot D_f \cdot N_q^* \cdot F_{qs}^*$$

$$F_{qs}^* := 1 - m_1 \cdot \left(\frac{B}{L}\right)$$

$$F_{\gamma s}^* := 1 - m_2 \cdot \left(\frac{B}{L}\right)$$

For clays ϕ=0

$$q_{u(\text{continuous})} := c_u \cdot N_c^* + \gamma \cdot D_f$$

No Nq

$$q_{u(\text{square})} := \underbrace{5.14 \cdot \left(1 + \frac{0.5 \cdot \frac{B}{H} - 0.707}{5.14}\right)}_{N_{c(\text{square})}^*} \cdot c_u + \gamma \cdot D_f$$

Table 4.1

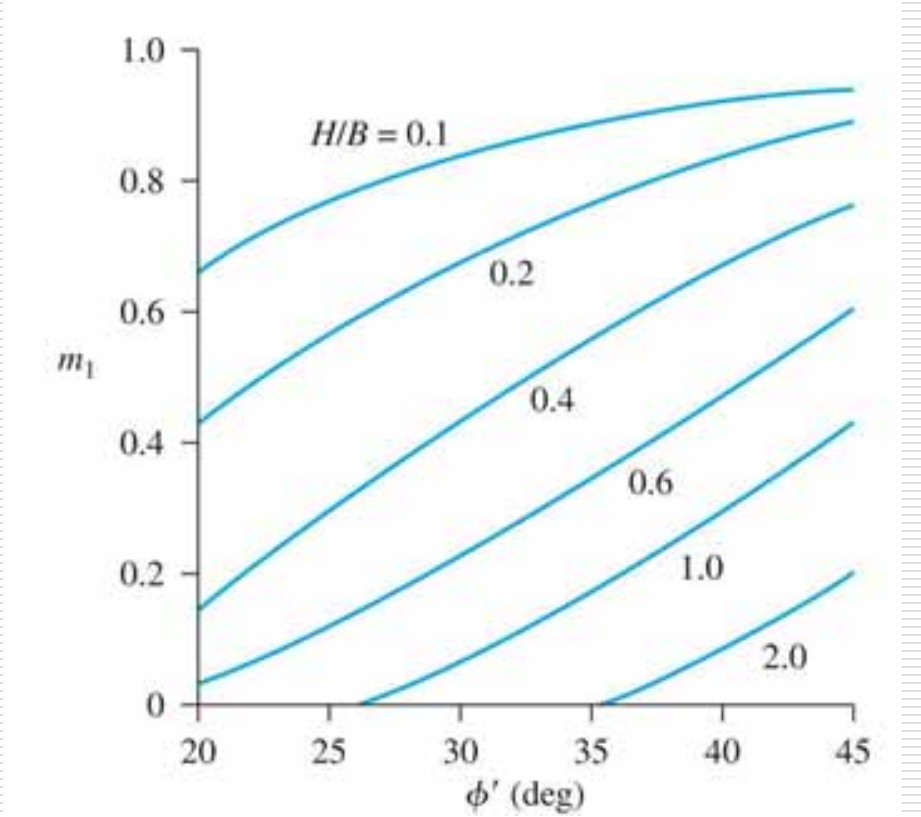
N*c for ϕ = 0

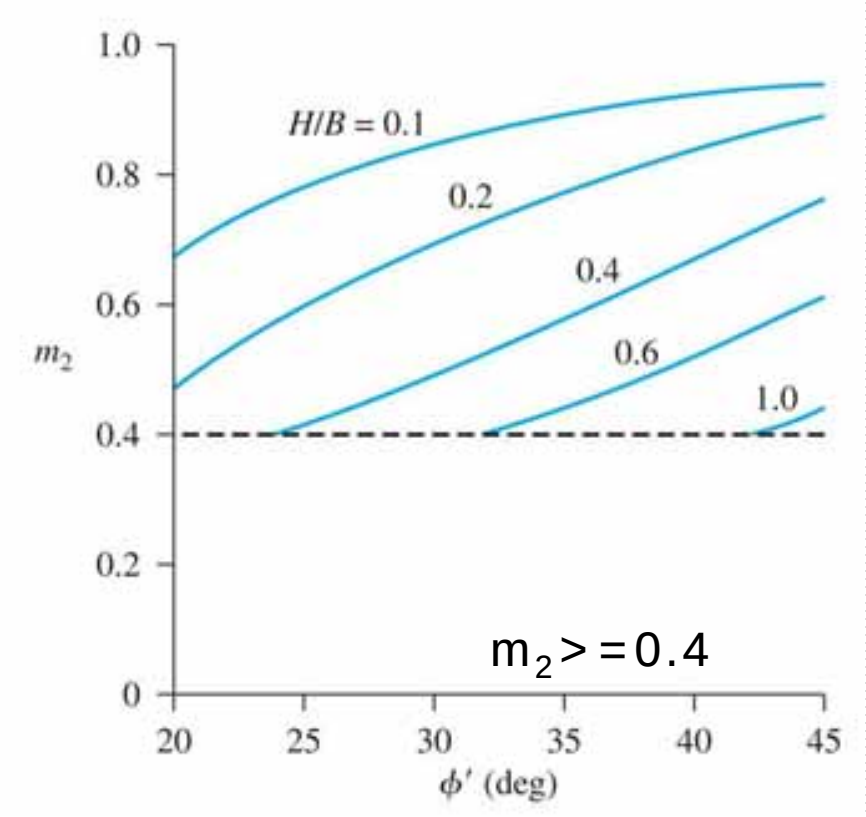
Table 4.1 Values of N_c^* for Continuous and Square Foundations ($\phi = 0$)

$\frac{B}{H}$	N_c^* Square[a]	N_c^* Continuous[b]
2	5.43	5.24
3	5.93	5.71
4	6.44	6.22
5	6.94	6.68
6	7.43	7.20
8	8.43	8.17
10	9.43	9.05

[a] Buisman's analysis (1940)
[b] Mandel and Salencon's analysis (1972)

Variation of m_1 and m_2 with H/B and ϕ'





Example – Square Footing Sand

ϕ = 30° D_f = 2 feet H=4 feet B=8 feet γ= 115 pcf

$$q_u := \frac{\gamma \cdot B \cdot N^*_\gamma \cdot F^*_{\gamma s}}{2} + \gamma \cdot D_f \cdot N^*_q \cdot F^*_{qs}$$

$$F^*_{qs} := 1 - m_1 \cdot \left(\frac{B}{L}\right)$$

$$F^*_{\gamma s} := 1 - m_2 \cdot \left(\frac{B}{L}\right)$$

H/B = 4/8 = 0.5 B/L = 1 m_1 = 0.3 m_2 = 0.44 F^*_{qs} = 0.7 $F^*_{\gamma s}$ = 0.56

N^*_γ = 20 N^*_q = 50

q_u = (0.5)(115)(8)(20)(0.56) + (115)(2)(50)(0.7) = 13202 psf

Weak Base

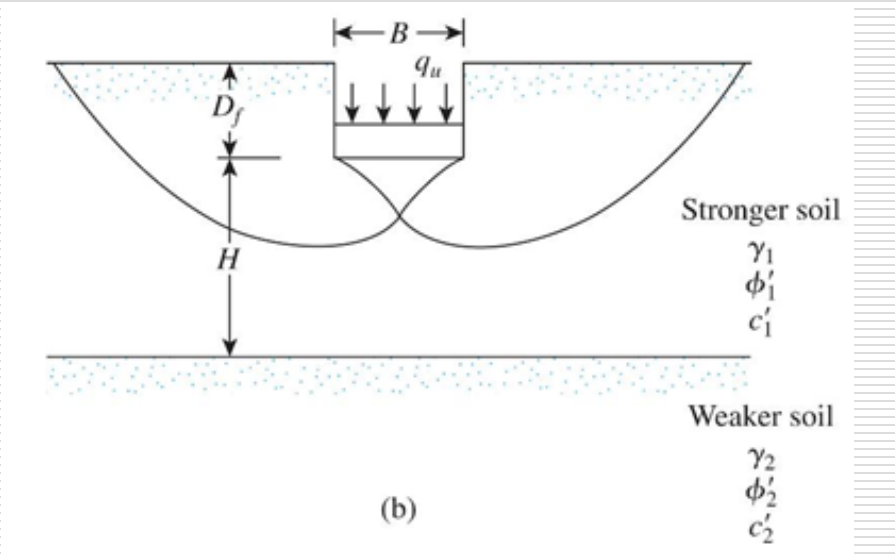
In some cases, the footing is founded on a strong soil underlain by a weaker soil. In this instance we must take into account the strength of both layers.

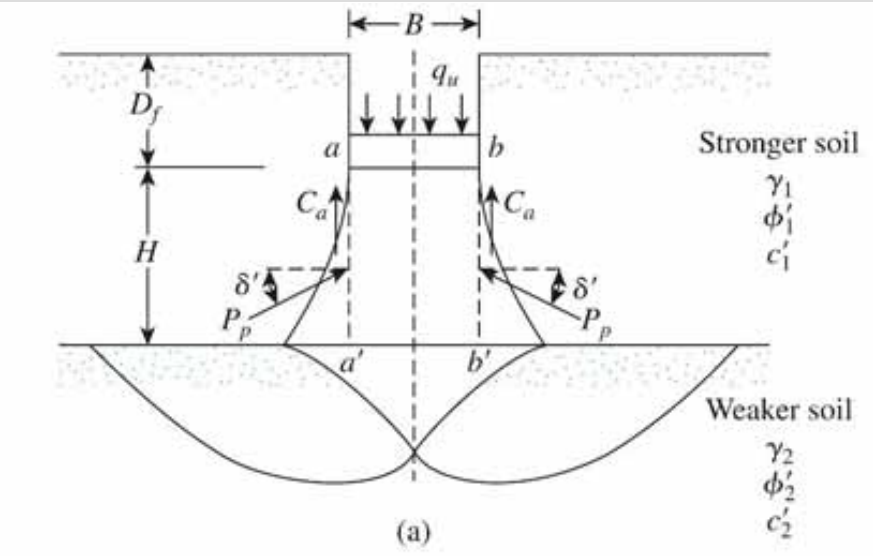
Layer	Unit Weight	Soil Friction Angle	Cohesion
Top	$\gamma 1$	$\phi'1$	c'1
Bottom	$\gamma 2$	$\phi'2$	c'2

Key – Pay attention to where each layers values are used in equations

Bearing capacity of a continuous foundation on layer soil

Resistance in upper layer + q_{bottom}





Punching shear failure in stronger soil with a general shear failure in the weak soil

Two forces resisting in upper layer
Passive earth pressure – P_p
Adhesion – Ca*H

Bearing Capacity for Each Layer

First – calculate bearing capacity for each layer without depth influence

Note: Continuous footing

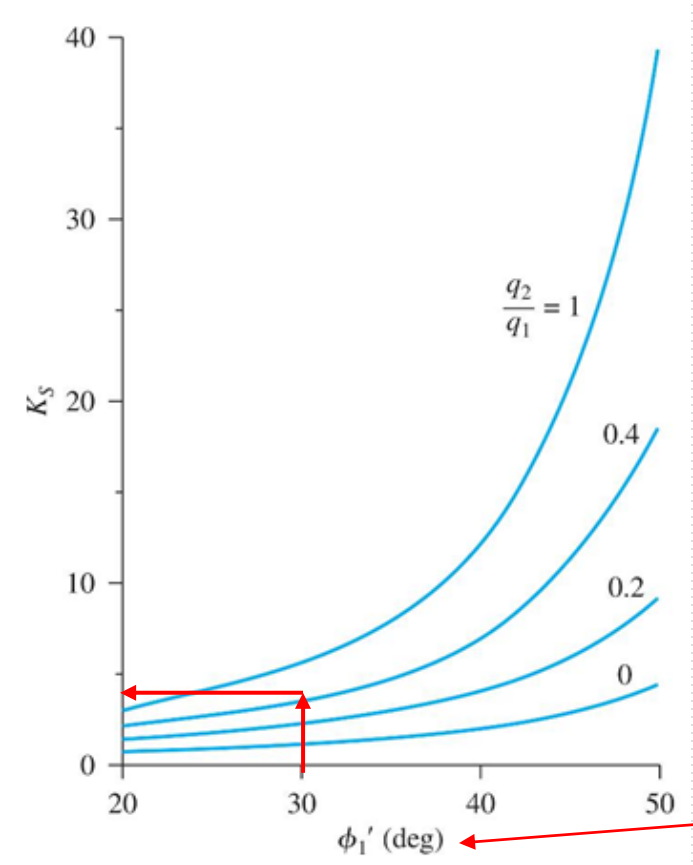
$$q_{top} = c'_1 \bullet N_{c(1)} + \tfrac{1}{2} \bullet \gamma_1 \bullet B \bullet N_{\gamma(1)}$$

For that layer

$$q_{bottom} = c'_2 \bullet N_{c(2)} + \tfrac{1}{2} \bullet \gamma_2 \bullet B \bullet N_{\gamma(2)}$$

Note: No q*Nq

Meyerhof and Hanna punching shear coefficient K_s



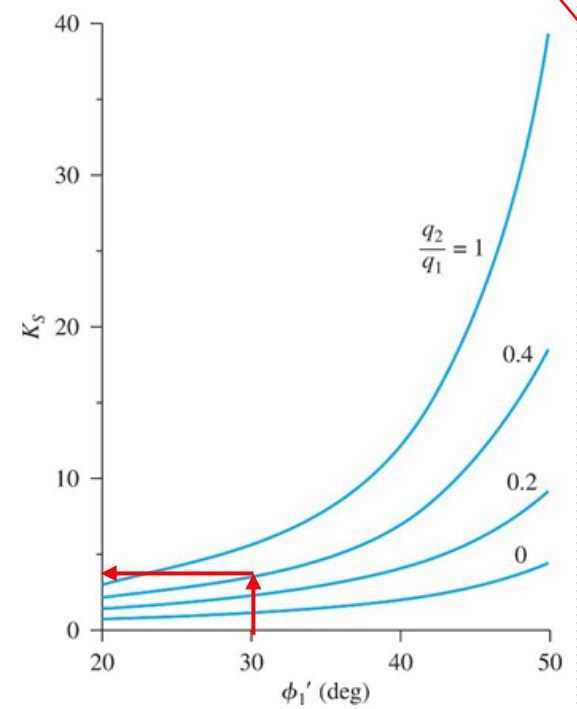
For this to apply
$q_2/q_1 <= 1$

Note: $q_1 = q_{top}$
$q_2 = q_{bottom}$

Friction angle of upper soil

Combined Bearing Capacity – Continuous Footing

$$q_u := q_{bottom} + \frac{2\,c'_a\,H}{B} + \gamma_1\,H^2\left(1+\frac{2\,D_f}{H}\right)\left(\frac{K_s\cdot\tan(\phi'_1)}{B}\right) - \gamma_1\,H \le q_{top}$$



Meyerhof and Hanna punching shear coefficient Ks

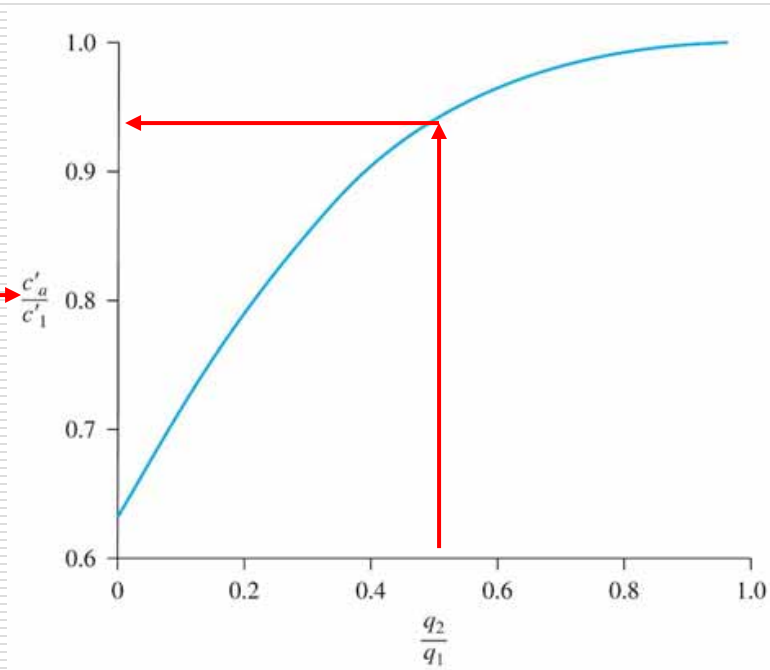


Figure 4.10 Variation of c'_a/c'_1 with q_2/q_1 based on the theory of Meyerhof and Hanna (1978)

Combined Bearing Capacity – Square Footing

$$q_u := q_{bottom} + \left(1 + \frac{B}{L}\right)\left(\frac{2\,c_a' H}{B}\right) + \gamma_1 H^2 \left(1 + \frac{B}{L}\right)\left(1 + \frac{2\,D_f}{H}\right)\left(\frac{1\,K_s \cdot \tan(\phi_1')}{B}\right) - \gamma_1 H \leq q_{top}$$

$$q_{bottom} := c_2' N_{c(2)} \cdot F_{cs(2)} + \gamma_1 \cdot (D_f + H) \cdot N_{q(2)} \cdot F_{qs(2)} + \frac{1}{2} \cdot \gamma_2 \cdot B \cdot N_{\gamma(2)} \cdot F_{\gamma s(2)}$$

For that layer

$$q_{top} := c_1 N_{c(1)} \cdot F_{cs(1)} + \gamma_1 \cdot D_f \cdot N_{q(1)} \cdot F_{qs(1)} + \frac{1}{2} \cdot \gamma_1 \cdot B \cdot N_{\gamma(1)} \cdot F_{\gamma s(1)}$$

Shape Factors from Chapter 3

Strong Sand Over Weak Clay

$$q_{bottom} := \left(1 + 0.2 \cdot \frac{B}{L}\right) 5.14\, c_2 + \gamma_1 \left(D_f + H\right)$$

Of top layer

$$q_{top} := \gamma_1 \cdot D_f\, N_{q(1)}\, F_{qs(1)} + \frac{1}{2}\, \gamma_1\, B \cdot N_{\gamma(1)}\, F_{\gamma s(1)}$$

$$q_u := \left(1 + \frac{0.2\, B}{L}\right) 5.14\, c_2' + \gamma_1\, H^2 \left(1 + \frac{B}{L}\right)\left(1 + \frac{2\, D_f}{H}\right)\left(\frac{K_s \cdot \tan(\phi_1)}{B}\right) + \gamma_1\, D_f \le q_{top}$$

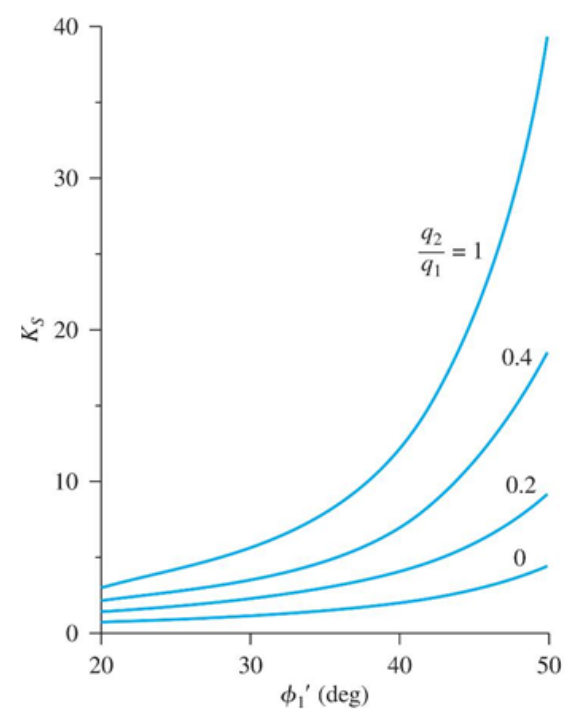
Of bottom layer

Of top layer

Ratio simplified to

$$\frac{q_2}{q_1} := \frac{5.14\, c_2}{0.5\, \gamma_1\, B \cdot N_{\gamma(1)}}$$

To determine Ks



Strong Sand Over Weak Sand

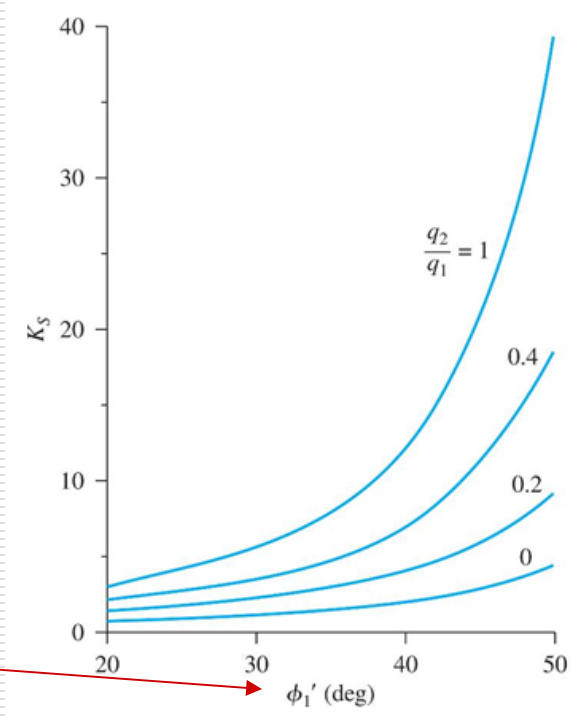
$$q_u := \left[\gamma_1 (D_f + H) N_{q(2)} F_{qs(2)} + \frac{1}{2} \cdot \gamma_2 B \cdot N_{\gamma(2)} F_{\gamma s(2)}\right] + \gamma_1 H^2 \left(1 + \frac{B}{L}\right)\left(1 + \frac{2 D_f}{H}\right)\left(\frac{K_{s \cdot \tan(\phi_1')}}{B}\right) - \gamma_1 H \leq q_{top}$$

where

$$q_{top} := \gamma_1 \cdot D_f N_{q(1)} F_{qs(1)} + \frac{1}{2} \gamma_1 B \cdot N_{\gamma(1)} F_{\gamma s(1)}$$

and Ratio simplified to

$$\frac{q_2}{q_1} := \frac{\gamma_2 N_{\gamma(2)}}{\gamma_1 N_{\gamma(1)}}$$



Strong Clay Over Weak Clay

$$q_u := \left(1 + \frac{0.2\,B}{L}\right) 5.14\,c_2 + \left(1 + \frac{B}{L}\right)\left[\frac{(2\,c_a\,H)}{B}\right] + \gamma_1\,D_f$$

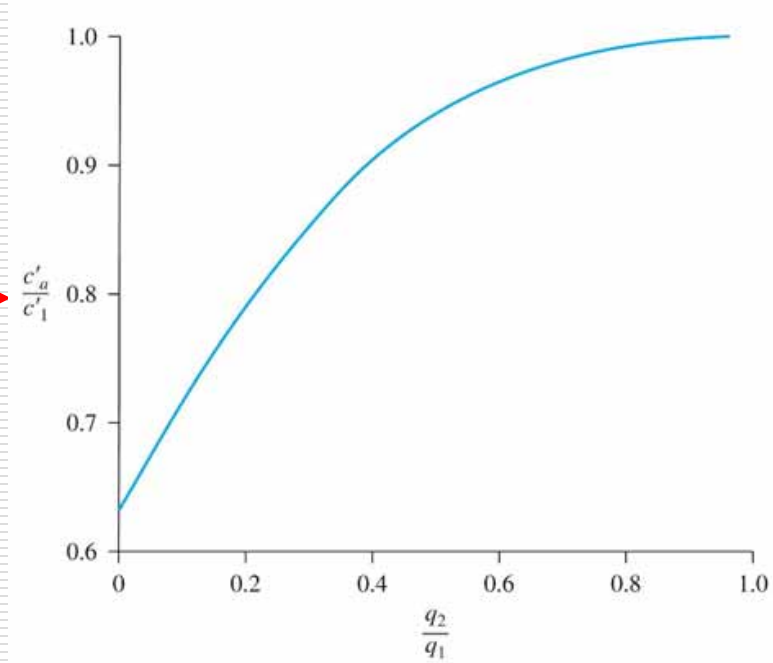
where

$$q_{top} := \left(1 + 0.2 \cdot \frac{B}{L}\right) 5.14\,c_1 + \gamma_1\,D_f$$

and Ratio simplified to

$$\frac{q_2}{q_1} := \frac{c_2}{c_1}$$

C_1 and c_2 are undrained



Critical Depth

At what depth does the lower layer impact foundation bearing capacity?

In all instances the equations are shown to be less than or equal to q_{top}. Set $q_u = q_{top}$ and solve for the only unknown – H.

Another estimate which assumes sand (c=0) over clay (ϕ = 0) (not in textbook)

$$H_c = \frac{3\,B \cdot \ln\left(\frac{q_{top}}{q_{bottom}}\right)}{2 \cdot \left(1 + \frac{B}{L}\right)}$$

For continuous, denominator =2
For square, denominator = 4
Therefore the critical depth for a continuous footing would be twice that for a square footing.

Critical Depth

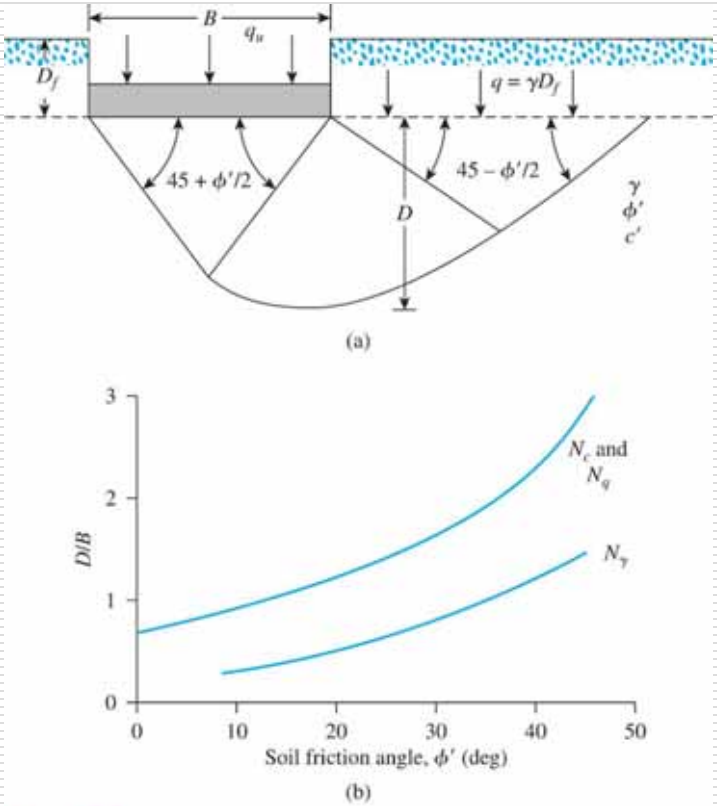
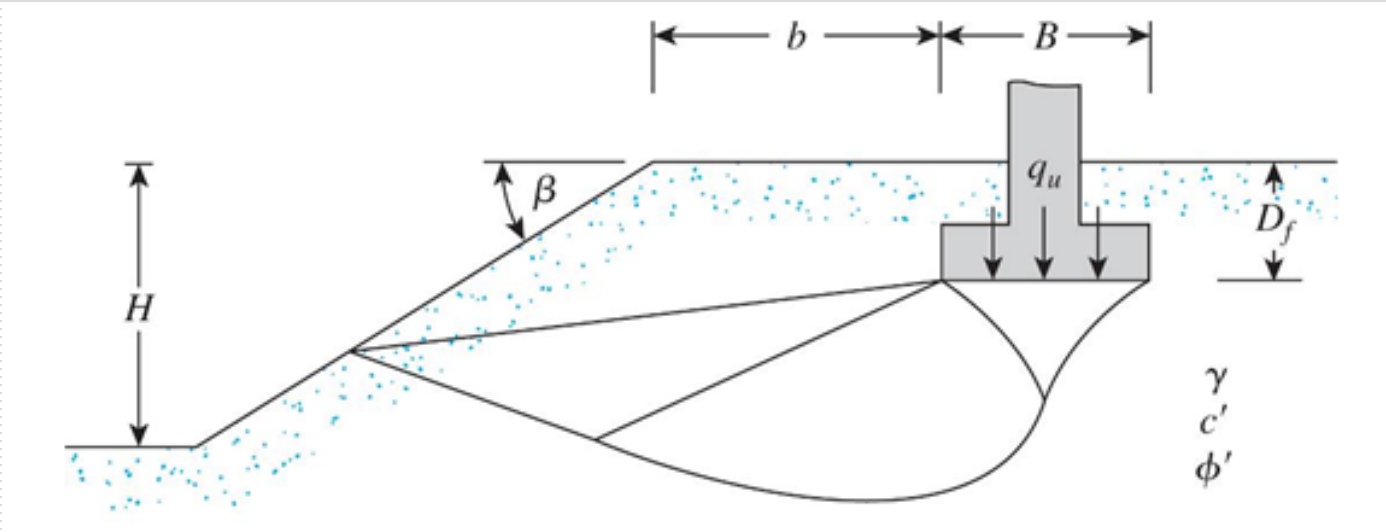


Figure 4.1 (a) Failure surface under a rough continuous foundation; (b) variation of D/B with soil friction angle ϕ'

Bearing Capacity on Slopes

Footings founded at the top of slopes can have the shear failure zone intersected by the slope. This changes the conditions within the failure zone and affects BC.



Bearing Capacity

$$q_u := \frac{1}{2}\gamma B N_{\gamma q} + \acute{c} N_{cq}$$

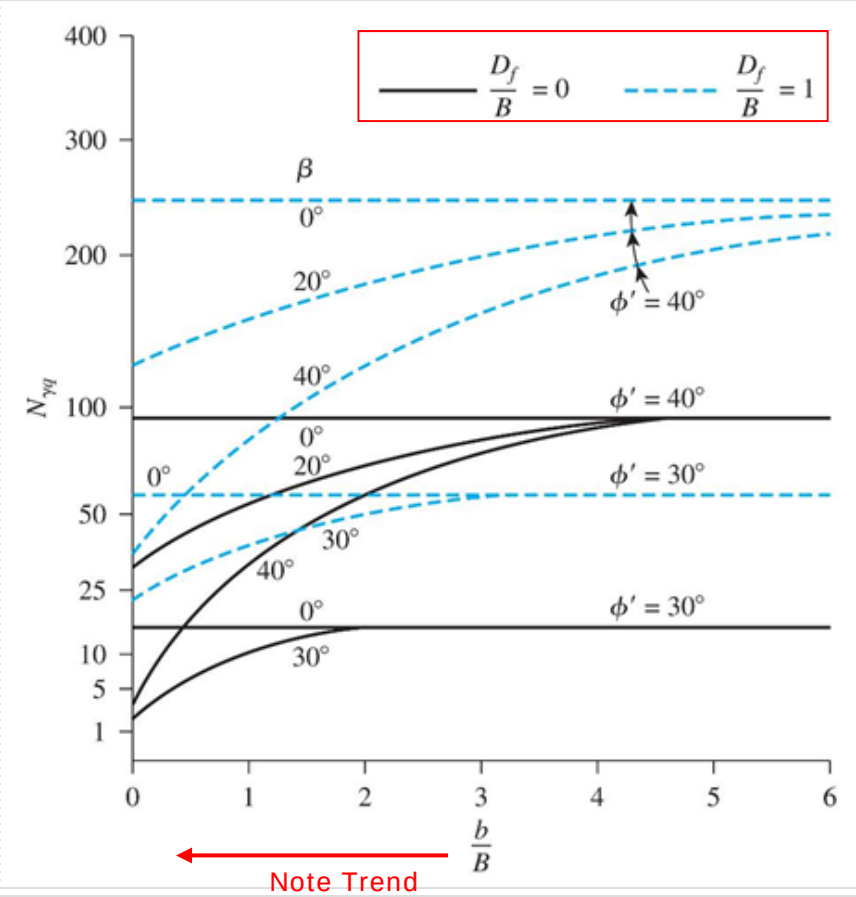
$$q_u := \frac{1}{2}\gamma B N_{\gamma q}$$ **For sands**

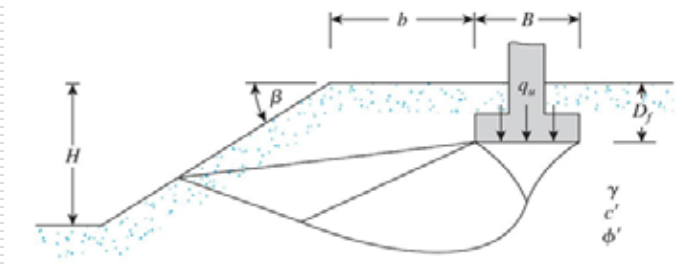
Note no q*Nq

$$q_u := \acute{c} N_{cq}$$ **For clays**

Obtain $N_{\gamma q}$ from Fig. 4.15 and N_{cq} from Fig. 4.16

Meyerhof's bearing capacity factor $N_{\gamma q}$ for granular soil ($c' = 0$)





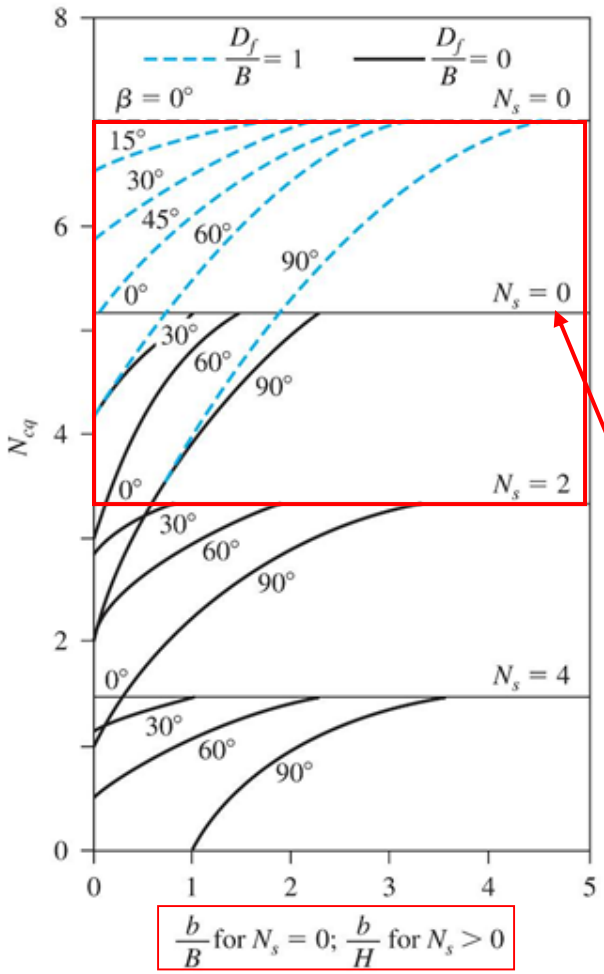
What do you do if $0 > D_f/B < 1$?
If $30 < \phi < 40$?

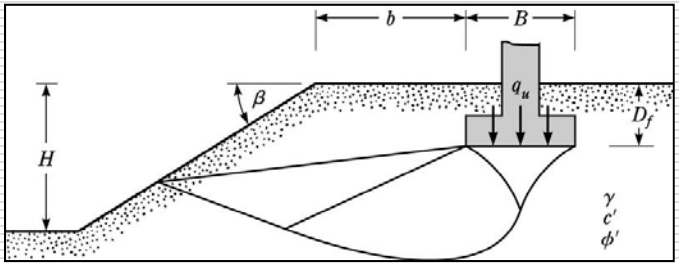
Stability Number for $\phi = 0$ Soils

$$N_s := \frac{\gamma H}{c}$$

If B<H,use the curves for $N_s = 0$
IF B>=H, use the curves for calculated N_s

Meyerhof's bearing capacity factor N_{cq} for purely cohesive soil





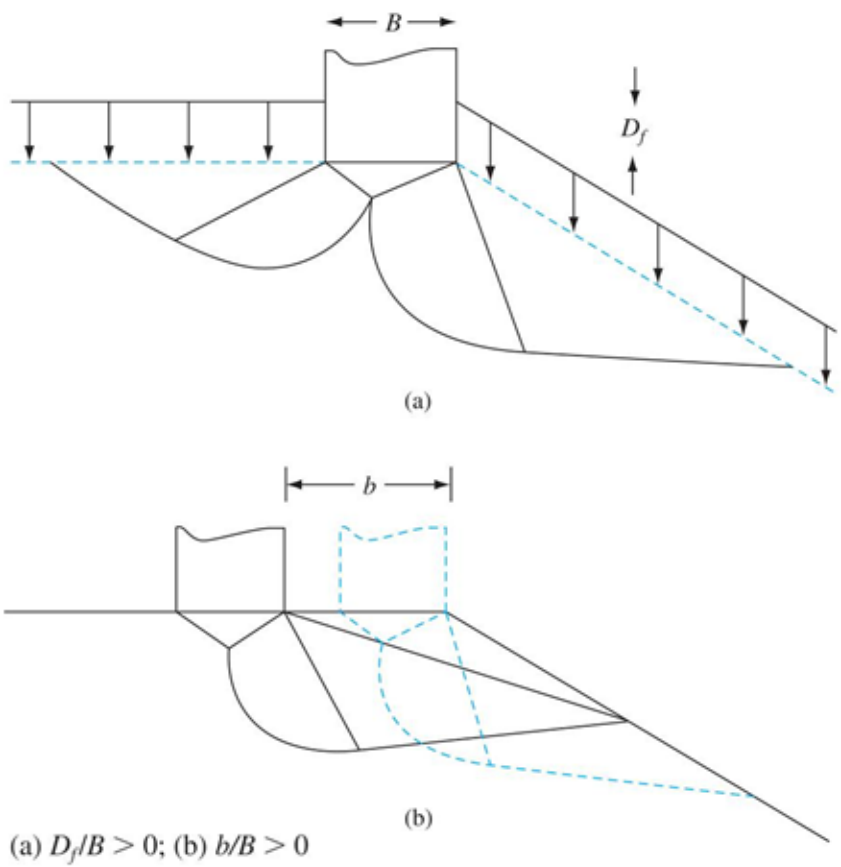
If B<H, use the curves for N_s=0
IF B>=H, use the curves for calculated N_s

Note: Ns <= 5.14 for D_f/B=0

Stress Characteristics Solution

$$q_u := \frac{1}{2}\gamma B N_{\gamma q}$$

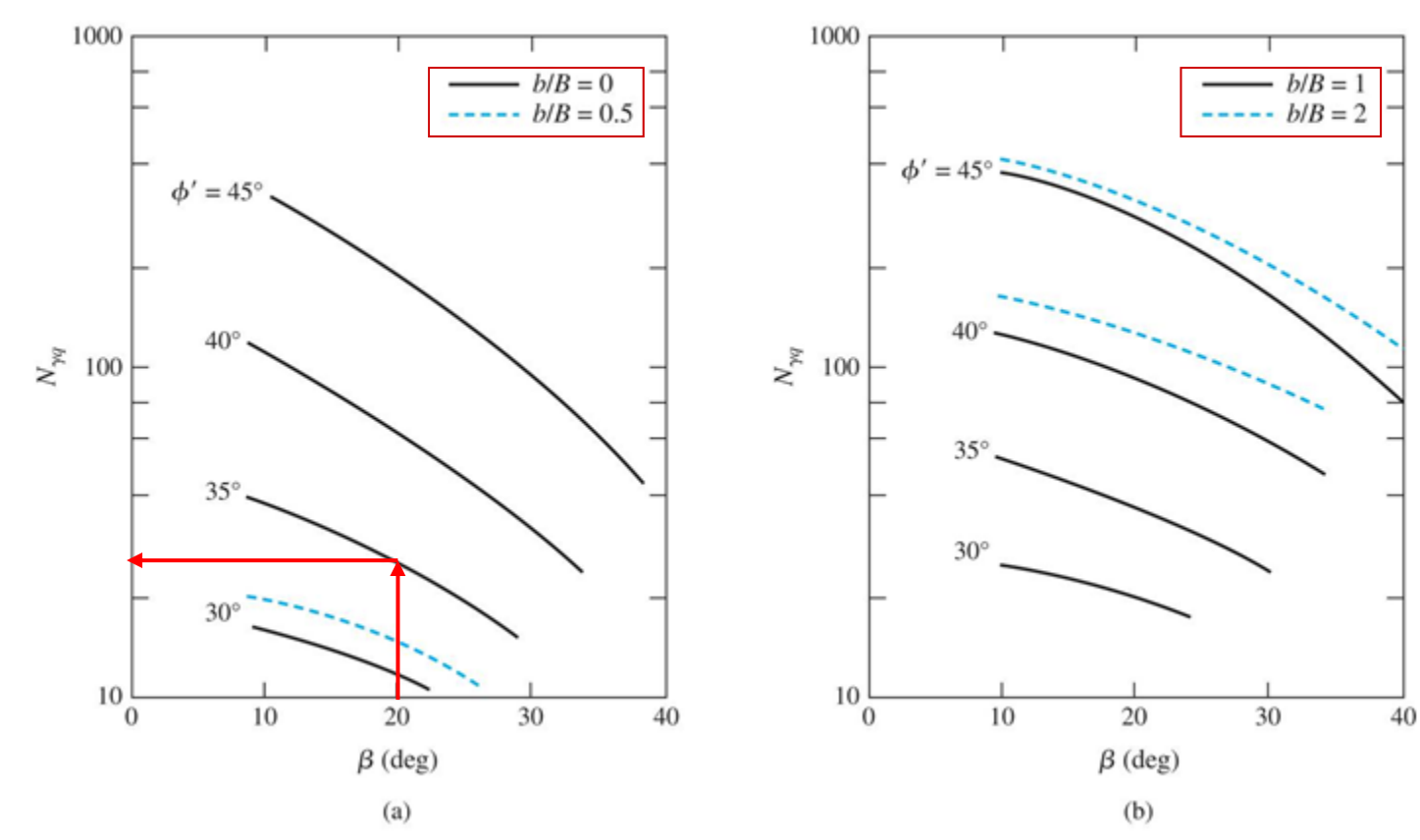
For granular soils c=0
Cannot use for cohesive soils



(a) $D_f/B > 0$; (b) $b/B > 0$

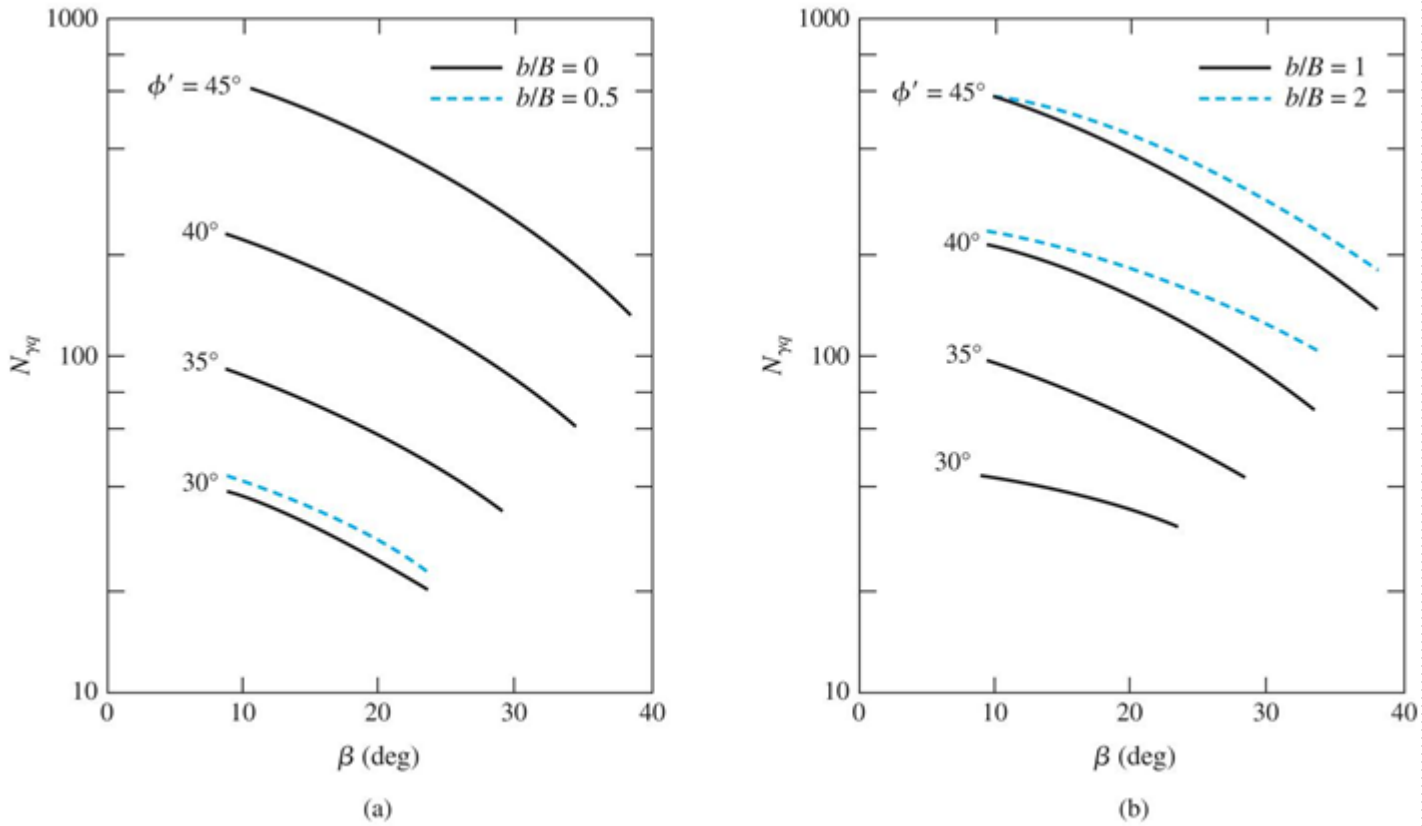
Graham et al.'s theoretical values of $N_{\gamma q}$ ($D_f/B = 0$)

b/B=0.5 ϕ=40° ? If no dashed line, use solid line



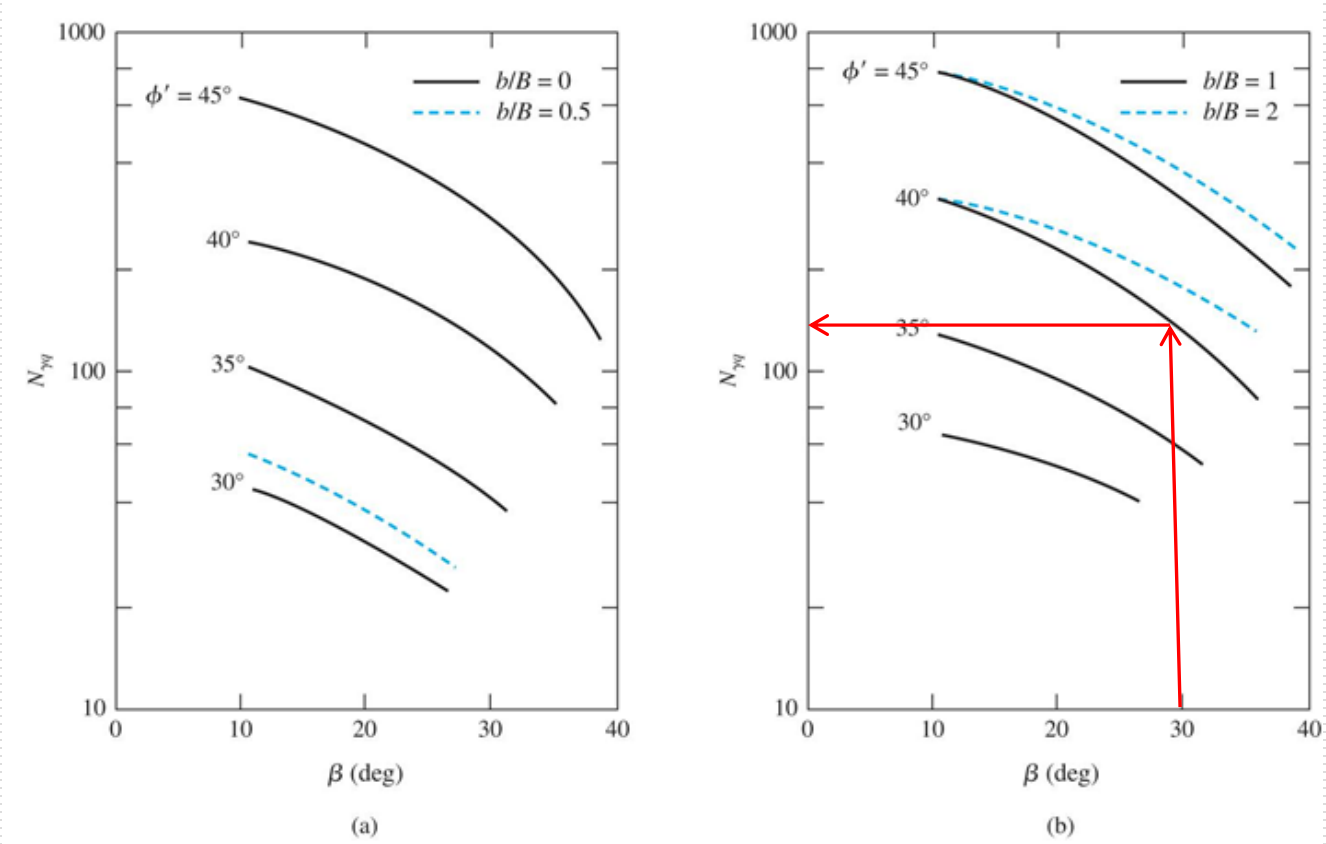
Graham et al.'s theoretical values of $N_{\gamma q}$ (D_f/B = 0.5)

If no dashed line, use solid line

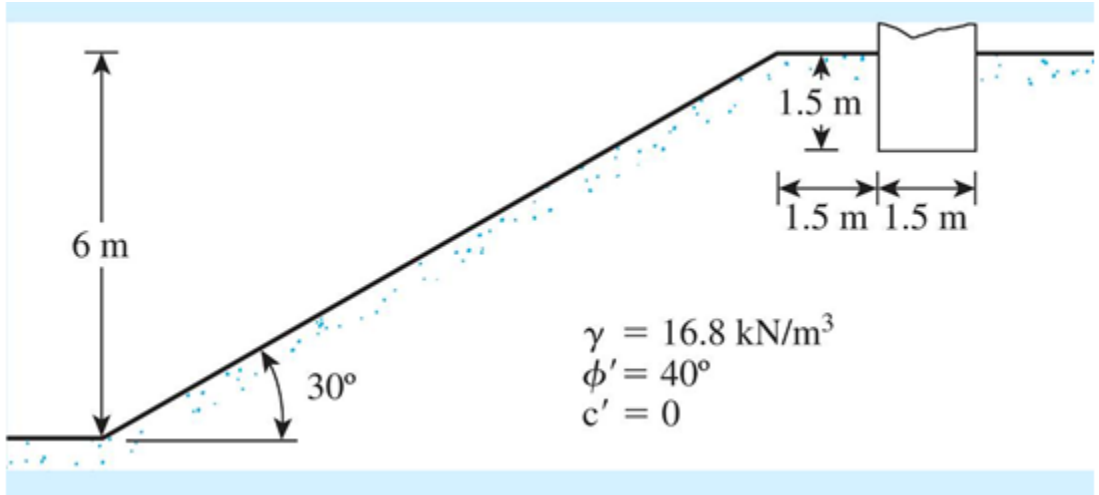


Graham et al.'s theoretical values of $N_{\gamma q}$ ($D_f/B = 1$)

If no dashed line, use solid line



Problem



For granular soil (c=0), use eq. 4.38

$$q_u = 1/2\gamma B N_{\gamma q}$$

b/B = 1.5/1.5 = 1 Df/B = 1.5/1.5 = 1 and ϕ = 40° and β = 30°

From Fig 4.15, $N_{\gamma q}$ = 120

$$q_u = 0.5(16.8)(1.5)(120) = 1512 \text{ kN/m}^2$$

Homework

From Chapter 4

CE 430

- 4.1
- 4.4
- 4.5
- 4.6
- 4.11

CE 530

Same as CE 430 plus

4.3

4.7

Read Chapter 5