Special Cases

All our analyses to this point have assumed the following:

- The soil supporting the foundation below its base is homogeneous and extends to great depth.
- The ground surface is horizontal.

This is not always the case.

- There will be cases where more than one soil type will be present within the failure zone. Such cases include:
  1. Rigid base that prevents full development of general shear
  2. Stronger soil underlain by weaker soil
- The foundation will be on or near a sloped ground surface.
(a) Failure surface under a rough continuous foundation; (b) variation of $D/B$ with soil friction angle $\phi'$
Rigid Base Failure Mode

Failure surface under a rough, continuous foundation with a rigid, rough base located at a shallow depth (ie. Rock)

\[ H \leq B \]
Rigid Base

$$q_u := \frac{\gamma \cdot B \cdot N^*}{2} + c' \cdot nN_c^* + \gamma \cdot D_f \cdot N_q^*$$

Same equation we have been using except we modify the bearing capacity factors based on the depth to the rigid base and its ratio with footing width.

Note: we do not take the strength of the rigid base into account in our analyses. It is assumed to be strong enough not to fail.
Mandel and Salencon’s bearing capacity factor $N^*_c$ (Equation 4.2)

$$N_c = 37$$
$$N_c^* = 80$$
Mandel and Salencon’s bearing capacity factor $N^*_q$ (Equation 4.2)

$N_q = 22$

$N_q^* = 50$
Mandel and Salencon’s bearing capacity factor $N^*_{\gamma}$ (Equation 4.2)

\[ N_{\gamma} = 19 \]
\[ N_{\gamma}^* = 20 \]
Two Simplified Cases

**For sands c=0** (Rectangular or Square) – What about continuous?

\[
q_u := \frac{\gamma \cdot B \cdot N^* \cdot F^*}{2} + \gamma \cdot D_f \cdot N^* \cdot F^*_q
\]

\[
F^*_qs := 1 - m_1 \cdot \left( \frac{B}{L} \right)
\]

**For clays \( \phi=0 \)**

\[
q_u(\text{continuous}) := c_u \cdot N^*_c + \gamma \cdot D_f
\]

\[
q_u(\text{square}) := 5.14 \left( 0.5 \cdot \frac{B}{H} - 0.707 \right) + \frac{B}{5.14} \cdot c_u + \gamma \cdot D_f
\]

No \( N_q \)

Table 4.1
N*c for $\phi = 0$

Table 4.1 Values of $N_c^*$ for Continuous and Square Foundations ($\phi = 0$)

<table>
<thead>
<tr>
<th>$\frac{B}{H}$</th>
<th>$N_c^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Square$^a$</td>
</tr>
<tr>
<td>2</td>
<td>5.43</td>
</tr>
<tr>
<td>3</td>
<td>5.93</td>
</tr>
<tr>
<td>4</td>
<td>6.44</td>
</tr>
<tr>
<td>5</td>
<td>6.94</td>
</tr>
<tr>
<td>6</td>
<td>7.43</td>
</tr>
<tr>
<td>8</td>
<td>8.43</td>
</tr>
<tr>
<td>10</td>
<td>9.43</td>
</tr>
</tbody>
</table>

$^a$Buisman’s analysis (1940)

$^b$Mandel and Salencon’s analysis (1972)
Variation of $m_1$ and $m_2$ with $H/B$ and $\phi'$

$m_2 \geq 0.4$
Example – Square Footing Sand

\( \phi = 30^\circ \) \( D_f = 2 \) feet \( H = 4 \) feet \( B = 8 \) feet \( \gamma = 115 \) pcf

\[
q_u := \frac{\gamma \cdot B \cdot N^*_\gamma \cdot F^*}{2} + \gamma \cdot D_f \cdot N^*_q \cdot F^*_q S
\]

\[
F^*_q s := 1 - m_1 \cdot \left( \frac{B}{L} \right)
\]

\[
F^*_\gamma s := 1 - m_2 \cdot \left( \frac{B}{L} \right)
\]

\[
\frac{H}{B} = \frac{4}{8} = 0.5 \quad \frac{B}{L} = 1 \quad m_1 = 0.3 \quad m_2 = 0.44 \quad F^*_q s = 0.7 \quad F^*_\gamma s = 0.56
\]

\[
N^*_\gamma = 20 \quad N^*_q = 50
\]

\[
q_u = (0.5)(115)(8)(20)(0.56) + (115)(2)(50)(0.7) = 13202 \text{ psf}
\]
Weak Base

In some cases, the footing is founded on a strong soil underlain by a weaker soil. In this instance we must take into account the strength of both layers.

Soil Friction

<table>
<thead>
<tr>
<th>Layer</th>
<th>Unit Weight</th>
<th>Angle</th>
<th>Cohesion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>$\gamma_1$</td>
<td>$\phi'1$</td>
<td>$c'1$</td>
</tr>
<tr>
<td>Bottom</td>
<td>$\gamma_2$</td>
<td>$\phi'2$</td>
<td>$c'2$</td>
</tr>
</tbody>
</table>

Key – Pay attention to where each layers values are used in equations
Bearing capacity of a continuous foundation on layer soil

Resistance in upper layer + $q_{\text{bottom}}$

Punching shear failure in stronger soil with a general shear failure in the weak soil

Two forces resisting in upper layer
Passive earth pressure – $P_p$
Adhesion – $C_a \times H$
Bearing Capacity for Each Layer

First – calculate bearing capacity for each layer without depth influence

Note: Continuous footing

\[ q_{\text{top}} = c'_{1} \cdot N_{c(1)} + \frac{1}{2} \cdot \gamma_{1} \cdot B \cdot N_{\gamma(1)} \]

\[ q_{\text{bottom}} = c'_{2} \cdot N_{c(2)} + \frac{1}{2} \cdot \gamma_{2} \cdot B \cdot N_{\gamma(2)} \]

Note: No q*Nq

For that layer
Meyerhof and Hanna punching shear coefficient $K_S$

For this to apply
$q_2/q_1 \leq 1$

Note: $q_1 = q_{\text{top}}$
$q_2 = q_{\text{bottom}}$

Friction angle of upper soil
Combined Bearing Capacity – Continuous Footing

\[ q_{u} := q_{\text{bottom}} + \frac{2 \, c_a' \, H}{B} + \gamma_{1} \, H^2 \left( 1 + \frac{2 \, D_f}{H} \right) \left( \frac{K_s \cdot \tan(\phi_1')}{B} \right) - \gamma_{1} \, H \leq q_{\text{top}} \]

Figure 4.9

Meyerhof and Hanna punching shear coefficient \( K_s \)

Figure 4.10

Variation of \( \frac{c_a' \, c_1'}{c_1'} \) with \( \frac{q_2}{q_1} \) based on the theory of Meyerhof and Hanna (1978)
Combined Bearing Capacity – Square Footing

\[ q_u := q_{\text{bottom}} + \left(1 + \frac{B}{L}\right) \left(\frac{2 c a H}{B}\right) + \gamma_1 H^2 \left(1 + \frac{B}{L}\right) \left(1 + \frac{2 D_f}{H}\right) \left(1 \cdot K_s\cdot \tan(\phi_1)\right) - \gamma_1 H \leq q_{\text{top}} \]

For that layer

\[ q_{\text{bottom}} := c_2 N_c(2) \cdot F_{cs(2)} + \gamma_1 \cdot (D_f + H) \cdot N_q(2) \cdot F_{qs(2)} + \frac{1}{2} \cdot \gamma_2 \cdot B \cdot N_\gamma(2) \cdot F_{\gamma s(2)} \]

\[ q_{\text{top}} := c_1 N_c(1) \cdot F_{cs(1)} + \gamma_1 \cdot D_f \cdot N_q(1) \cdot F_{qs(1)} + \frac{1}{2} \cdot \gamma_1 \cdot B \cdot N_\gamma(1) \cdot F_{\gamma s(1)} \]

Shape Factors from Chapter 3
Strong Sand Over Weak Clay

\[ q_{\text{bottom}} := \left(1 + 0.2 \cdot \frac{B}{L}\right) 5.14 c_2 + \gamma_1 \left(D_f + H\right) \]

\[ q_{\text{top}} := \gamma_1 \cdot D_f \cdot \mathbb{N}_q(1) \cdot F_{qs}(1) + \frac{1}{2} \gamma_1 B \cdot \mathbb{N}_\gamma(1) \cdot F_{\gamma_s}(1) \]

\[ q_u := \left(1 + \frac{0.2 B}{L}\right) 5.14 c_2 + \gamma_1 H^2 \left(1 + \frac{B}{L}\right) \left(1 + \frac{2 D_f}{H}\right) \left(\frac{K_s \cdot \tan(\phi'_1)}{B}\right) + \gamma_1 D_f \]

Ratio simplified to

\[ \frac{q_2}{q_1} := \frac{5.14 c_2}{0.5 \gamma_1 B \cdot \mathbb{N}_\gamma(1)} \quad \text{To determine } K_s \]
**Strong Sand Over Weak Sand**

\[ q_{u} := \left[ \gamma_{1} (D_{f} + H) N_{q(2)} F_{qs(2)} + \frac{1}{2} \gamma_{2} B \cdot N_{\gamma(2)} F_{\gamma s(2)} \right] + \gamma_{1} H^{2} \left( 1 + \frac{B}{L} \right) \left( 1 + \frac{2 D_{f}}{H} \right) \left( \frac{K_{s} \cdot \tan(\phi_{1}^{'})}{B} \right) - \gamma_{1} H \leq q_{\text{top}} \]

**where**

\[ q_{\text{top}} := \gamma_{1} \cdot D_{f} \cdot N_{q(1)} \cdot F_{qs(1)} + \frac{1}{2} \gamma_{1} \cdot B \cdot N_{\gamma(1)} \cdot F_{\gamma s(1)} \]

**and** Ratio simplified to

\[ \frac{q_{2}}{q_{1}} := \frac{\gamma_{2} N_{\gamma(2)}}{\gamma_{1} N_{\gamma(1)}} \]
Strong Clay Over Weak Clay

\[ q_{u} := \left(1 + \frac{0.2 \cdot B}{L}\right) 5.14 c_2 + \left(1 + \frac{B}{L}\right) \left[ \frac{\left(2 c_a H\right)}{B}\right] + \gamma_1 D_f \]

where

\[ q_{\text{top}} := \left(1 + 0.2 \cdot \frac{B}{L}\right) 5.14 c_1 + \gamma_1 D_f \]

and  

\[ \frac{q_2}{q_1} := \frac{c_2}{c_1} \]

\(c_1\) and \(c_2\) are undrained
Critical Depth

At what depth does the lower layer impact foundation bearing capacity?

In all instances the equations are shown to be less than or equal to $q_{\text{top}}$. Set $q_u = q_{\text{top}}$ and solve for the only unknown – $H$.

Another estimate which assumes sand ($c=0$) over clay ($\phi = 0$) (not in textbook)

$$H_c = \frac{3 B \ln \left( \frac{q_{\text{top}}}{q_{\text{bottom}}} \right)}{2 \left( 1 + \frac{B}{L} \right)}$$

For continuous, denominator $= 2$
For square, denominator $= 4$
Therefore the critical depth for a continuous footing would be twice that for a square footing.
Critical Depth

Figure 4.1 (a) Failure surface under a rough continuous foundation; (b) variation of $D/B$ with soil friction angle $\phi'$
Bearing Capacity on Slopes

Footings founded at the top of slopes can have the shear failure zone intersected by the slope. This changes the conditions within the failure zone and affects BC.
Bearing Capacity

\[ q_u := \frac{1}{2} \gamma B N_{\gamma q} + c N_{cq} \]

For sands

\[ q_u := \frac{1}{2} \gamma B N_{\gamma q} \]

Note no \( q \times N_q \)

For clays

\[ q_u := c N_{cq} \]

Obtain \( N_{\gamma q} \) from Fig. 4.15 and \( N_{cq} \) from Fig. 4.16
Meyerhof’s bearing capacity factor $N_{q}$ for granular soil ($c' = 0$)

What do you do if $0 > D_f/B < 1$?
If $30 < \phi < 40$?
Stability Number for $\phi=0$ Soils

$$N_s := \frac{\gamma H}{c}$$

If $B<H$, use the curves for $N_s=0$

IF $B \geq H$, use the curves for calculated $N_s$
Meyerhof’s bearing capacity factor $N_{cq}$ for purely cohesive soil

If $B < H$, use the curves for $N_s = 0$
IF $B > H$, use the curves for calculated $N_s$

Note: $N_s \leq 5.14$ for $D_f/B=0$
Stress Characteristics
Solution

\[ q_u := \frac{1}{2} \gamma B N \gamma q \]

For granular soils \( c=0 \)
Cannot use for cohesive soils

(a) \( D_f/B > 0 \); (b) \( b/B > 0 \)
Graham et al.'s theoretical values of $N_{\gamma q}$ ($D_f/B = 0$)

If no dashed line, use solid line

- $b/B = 0.5$ Φ = 40°
- $\phi = 40^\circ$

(a) $\phi' = 45^\circ$
- $40^\circ$
- $35^\circ$
- $30^\circ$

(b) $\phi' = 45^\circ$
- $40^\circ$
- $35^\circ$
- $30^\circ$
Graham et al.'s theoretical values of $N_{\gamma q}$ ($D_f/B = 0.5$)

If no dashed line, use solid line
Graham et al.’s theoretical values of $N_{\gamma q}$ ($D_f/B = 1$)

If no dashed line, use solid line
For granular soil (c=0), use eq. 4.38

\[ q_u = \frac{1}{2} \gamma B N_{rq} \]

\[ b/B = 1.5/1.5 = 1 \quad D_f/B = 1.5/1.5 = 1 \quad \phi' = 40^\circ \quad \beta = 30^\circ \]

From Fig 4.15, \( N_{rq} = 120 \)

\[ q_u = 0.5(16.8)(1.5)(120) = 1512 \text{ kN/m}^2 \]
Homework

From Chapter 4

CE 430

☐ 4.1
☐ 4.4
☐ 4.5
☐ 4.6
☐ 4.11

CE 530

Same as CE 430 plus

☐ 4.11

4.3
4.7

Read Chapter 5