
Principles of Foundation Engineering

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Chapter 5

Shallow Foundations: Allowable Bearing Capacity And Settlement

Bearing Capacity

Ultimate bearing capacity is the maximum pressure a foundation can exert on the soil before a large scale failure occurs. Such failures can be catastrophic with collapse of the structure.

Allowable bearing capacity has been defined as the ultimate bearing capacity divided by an adequate safety factor to prevent a large scale "catastrophic" failure from occurring.

Due to our knowledge of soil mechanics, such failures rarely occur today unless grossly inadequate exploration of subsurface conditions was performed.

$q_{\text{allowable}}$ settlement

However is a foundation that has adequate safety against a catastrophic failure performing properly if movement is sufficient to cause movements that is considered as damage to the structure?

Allowable bearing capacity is also the pressure where settlements will not create excessive movements that cause damage.

$$q_{\text{allowable}} = q_u / FS \text{ or } q_{\text{allowable settlement}}$$

whichever is smaller. Rarely will $q_{\text{allowable settlement}}$ be greater than q_u / FS unless FS too high.

Types of Settlement

Three types of settlement to consider:

- Elastic (immediate) – classic stress strain
- Primary consolidation
- Secondary consolidation

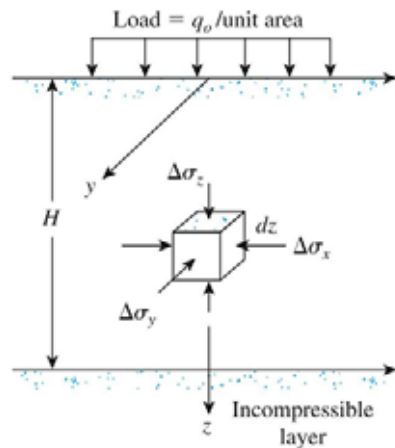
Elastic settlements occur in sands and in unsaturated fine grained soils (?).

Consolidation settlements occur in saturated fine grained soils. Remember consolidation theory is based on the soil being 100% saturated. Also permeability is sufficiently low that the stress is carried initially by the water and is gradually transferred to the soil as the pore water moves out.

Stress Creates Settlement

- A change in stress beyond what a soil is currently subject to σ' causes the soil to change properties:
 - Density
 - Strength and compressibility
 - Moisture Content
- The change in stress, $\Delta\sigma$, is estimated from the new loading q_0 and influence factor I : $\Delta\sigma = q_0(I)$
- Several methods for determining I .

Elastic settlement of shallow foundation



$$S_e = \int_0^H \epsilon_z dz = \frac{1}{E_s} \int_0^H (\Delta\sigma_z - \mu_s \Delta\sigma_x - \mu_s \Delta\sigma_y) dz$$

where

S_e = elastic settlement

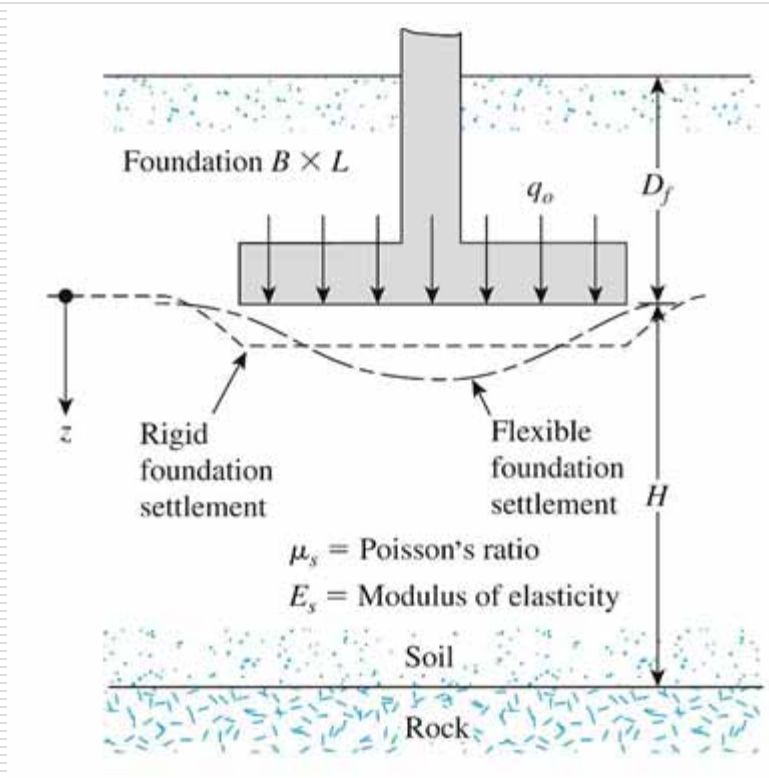
E_s = modulus of elasticity of soil

H = thickness of the soil layer

μ_s = Poisson's ratio of the soil

$\Delta\sigma_x, \Delta\sigma_y, \Delta\sigma_z$ = stress increase due to the net applied foundation load in the x , y , and z directions, respectively

Elastic settlement of flexible and rigid foundations



Examples

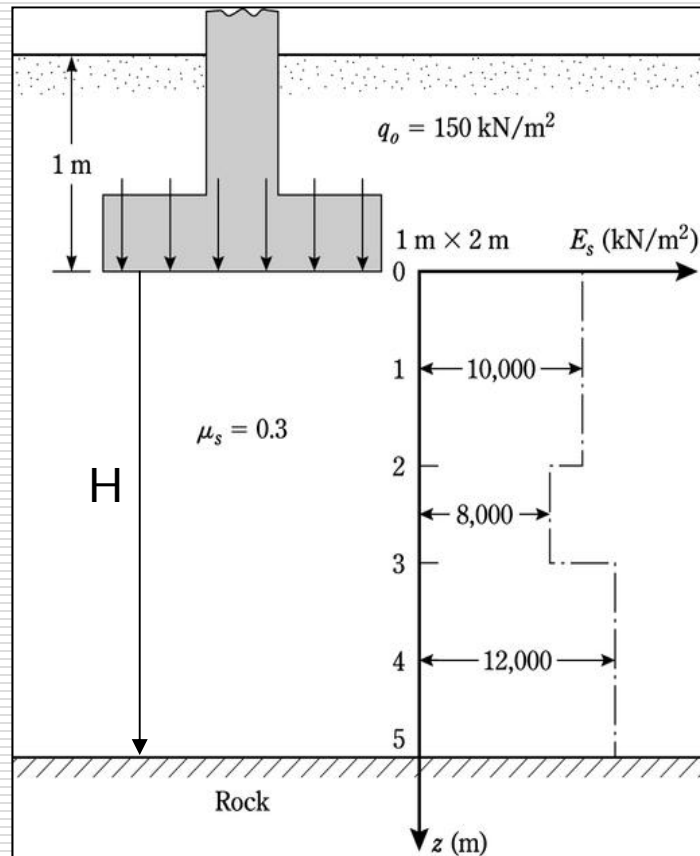
Flexible Foundation
Circular Fuel Storage Tank

Rigid Foundation
Concrete Building Foundation

Soils in Layers

- ❑ Subsurface conditions have layers.
- ❑ Soils are not homogeneous and isotropic even within a single layer.
- ❑ Not only do we need to determine the change in stress, $\Delta\sigma$, that a soil layer experiences, we need to evaluate the change in soil properties.
- ❑ Even then, we simplify. The amount of simplification affects accuracy.

Elastic settlement below the center of a foundation



$$\text{Strain} := \frac{\text{Stress}}{E}$$

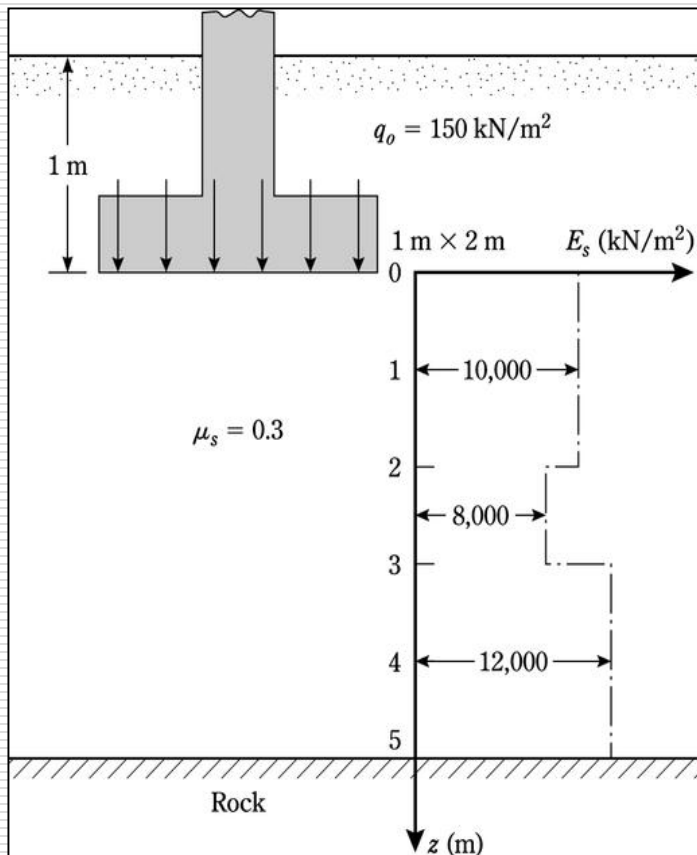
$$\text{Deformation} = \text{Strain} \times \text{Thickness}$$

Bowles Weighted Average

$$E_s = \frac{\sum (E_{s(i)} \cdot \Delta z)}{\bar{z}}$$

where $E_{s(i)}$ = soil modulus of elasticity within a depth Dz
 $\bar{z} = H$ or $5B$ whichever is smaller

Example Of Bowles Method



$$E_s = (10000(2) + 8000(1) + 12000(2))/5$$

$$E_s = 10400 \text{ kN/m}^2$$

$$\text{Strain} = 150/10400 = 0.014$$

$$\text{Settlement} = 0.014(5) = 0.07 \text{ m} = 7 \text{ cm}$$

This assumes no dissipation of stress with depth.

Settlement Based on Elastic Theory

$$S_e := q_o \cdot (\alpha \cdot B') \cdot \frac{(1 - \mu^2)}{E_s} \cdot I_s \cdot I_f$$

Where:

q_o = net applied pressure of foundation

μ = Poisson's ratio of the soil

E_s = average elastic modulus of soil from
 $z = 0$ to $z = 5B$

$B' = B/2$ for center of foundation & B for
corner

I_s = shape factor

I_f = depth factor

$$I_s := F_1 + \frac{(1 - 2\mu)}{(1 - \mu)} \cdot F_2$$

Get F_1 and F_2 from
Tables 5.8 & 5.9
using n & m

For center of footing

$$\alpha = 4$$

$$m := \frac{L}{B}$$

$$n := \frac{H}{\left(\frac{B}{2}\right)}$$

For corner of footing

$$\alpha = 1$$

$$m := \frac{L}{B}$$

$$n := \frac{H}{B}$$

Get I_f from Table 5.10

Determining F_1

Table 5.0 (Continued)

n'	m'									
	4.5	5.0	6.0	7.0	8.0	9.0	10.0	25.0	50.0	100.0
0.25	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
0.50	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036
0.75	0.073	0.073	0.072	0.072	0.072	0.072	0.071	0.071	0.071	0.071
1.00	0.114	0.113	0.112	0.112	0.112	0.111	0.111	0.110	0.110	0.110
1.25	0.155	0.154	0.153	0.152	0.152	0.151	0.151	0.150	0.150	0.150
1.50	0.195	0.194	0.192	0.191	0.190	0.190	0.189	0.188	0.188	0.188
1.75	0.233	0.232	0.229	0.228	0.227	0.226	0.225	0.223	0.223	0.223
2.00	0.269	0.267	0.264	0.262	0.261	0.260	0.259	0.257	0.256	0.256
2.25	0.302	0.300	0.296	0.294	0.293	0.291	0.291	0.287	0.287	0.287
2.50	0.333	0.331	0.327	0.324	0.322	0.321	0.320	0.316	0.315	0.315
2.75	0.362	0.359	0.355	0.352	0.350	0.348	0.347	0.343	0.342	0.342
3.00	0.389	0.386	0.382	0.378	0.376	0.374	0.373	0.368	0.367	0.367
3.25	0.415	0.412	0.407	0.403	0.401	0.399	0.397	0.391	0.390	0.390
3.50	0.438	0.435	0.430	0.427	0.424	0.421	0.420	0.413	0.412	0.411
3.75	0.461	0.458	0.453	0.449	0.446	0.443	0.441	0.433	0.432	0.432
4.00	0.482	0.479	0.474	0.470	0.466	0.464	0.462	0.453	0.451	0.451
4.25	0.516	0.496	0.484	0.473	0.471	0.471	0.470	0.468	0.462	0.460
4.50	0.520	0.517	0.513	0.508	0.505	0.502	0.499	0.489	0.487	0.487
4.75	0.537	0.535	0.530	0.526	0.523	0.519	0.517	0.506	0.504	0.503
5.00	0.554	0.552	0.548	0.543	0.540	0.536	0.534	0.522	0.519	0.519
5.25	0.569	0.568	0.564	0.560	0.556	0.553	0.550	0.537	0.534	0.534
5.50	0.584	0.583	0.579	0.575	0.571	0.568	0.585	0.551	0.549	0.548
5.75	0.597	0.597	0.594	0.590	0.586	0.583	0.580	0.565	0.583	0.562
6.00	0.611	0.610	0.608	0.604	0.601	0.598	0.595	0.579	0.576	0.575
6.25	0.623	0.623	0.621	0.618	0.615	0.611	0.608	0.592	0.589	0.588
6.50	0.635	0.635	0.634	0.631	0.628	0.625	0.622	0.605	0.601	0.600
6.75	0.646	0.647	0.646	0.644	0.641	0.637	0.634	0.617	0.613	0.612
7.00	0.656	0.658	0.658	0.656	0.653	0.650	0.647	0.628	0.624	0.623
7.25	0.666	0.669	0.669	0.668	0.665	0.662	0.659	0.640	0.635	0.634
7.50	0.676	0.679	0.680	0.679	0.676	0.673	0.670	0.651	0.646	0.645
7.75	0.685	0.688	0.690	0.689	0.687	0.684	0.681	0.661	0.656	0.655
8.00	0.694	0.697	0.700	0.700	0.698	0.695	0.692	0.672	0.666	0.665
8.25	0.702	0.706	0.710	0.710	0.708	0.705	0.703	0.682	0.676	0.675
8.50	0.710	0.714	0.719	0.719	0.718	0.715	0.713	0.692	0.686	0.684
8.75	0.717	0.722	0.727	0.728	0.727	0.725	0.723	0.701	0.695	0.693
9.00	0.725	0.730	0.736	0.737	0.736	0.735	0.732	0.710	0.704	0.702
9.25	0.731	0.737	0.744	0.746	0.745	0.744	0.742	0.719	0.713	0.711
9.50	0.738	0.744	0.752	0.754	0.754	0.753	0.751	0.728	0.721	0.719
9.75	0.744	0.751	0.759	0.762	0.762	0.761	0.759	0.737	0.729	0.727
10.00	0.750	0.758	0.766	0.770	0.770	0.770	0.768	0.745	0.738	0.735
20.00	0.878	0.896	0.925	0.945	0.959	0.969	0.977	0.982	0.965	0.957
50.00	0.962	0.989	1.034	1.070	1.100	1.125	1.146	1.265	1.279	1.261
100.00	0.990	1.020	1.072	1.114	1.150	1.182	1.209	1.408	1.489	1.499

Determining F_2

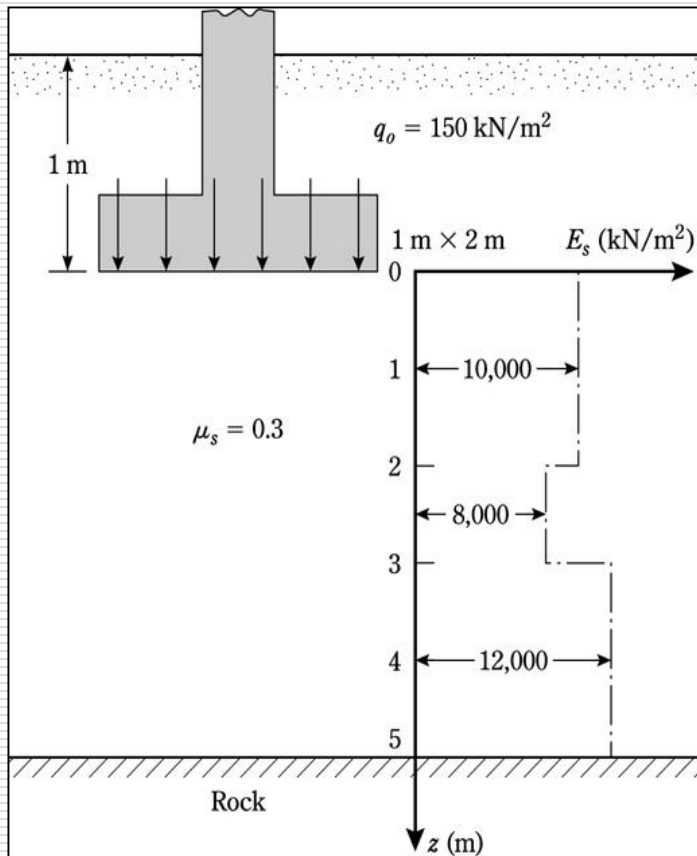
Table 5.9 Variation of F_2 with m' and n'

n'	m'									
	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	3.5	4.0
0.25	0.049	0.050	0.051	0.051	0.051	0.052	0.052	0.052	0.052	0.052
0.50	0.074	0.077	0.080	0.081	0.083	0.084	0.086	0.086	0.0878	0.087
0.75	0.083	0.089	0.093	0.097	0.099	0.101	0.104	0.106	0.107	0.108
1.00	0.083	0.091	0.098	0.102	0.106	0.109	0.114	0.117	0.119	0.120
1.25	0.080	0.089	0.096	0.102	0.107	0.111	0.118	0.122	0.125	0.127
1.50	0.075	0.084	0.093	0.099	0.105	0.110	0.118	0.124	0.128	0.130
1.75	0.069	0.079	0.088	0.095	0.101	0.107	0.117	0.123	0.128	0.131
2.00	0.064	0.074	0.083	0.090	0.097	0.102	0.114	0.121	0.127	0.131
2.25	0.059	0.069	0.077	0.085	0.092	0.098	0.110	0.119	0.125	0.130
2.50	0.055	0.064	0.073	0.080	0.087	0.093	0.106	0.115	0.122	0.127
2.75	0.051	0.060	0.068	0.076	0.082	0.089	0.102	0.111	0.119	0.125
3.00	0.048	0.056	0.064	0.071	0.078	0.084	0.097	0.108	0.116	0.122
3.25	0.045	0.053	0.060	0.067	0.074	0.080	0.093	0.104	0.112	0.119
3.50	0.042	0.050	0.057	0.064	0.070	0.076	0.089	0.100	0.109	0.116
3.75	0.040	0.047	0.054	0.060	0.067	0.073	0.086	0.096	0.105	0.113
4.00	0.037	0.044	0.051	0.057	0.063	0.069	0.082	0.093	0.102	0.110
4.25	0.036	0.042	0.049	0.055	0.061	0.066	0.079	0.090	0.099	0.107
4.50	0.034	0.040	0.046	0.052	0.058	0.063	0.076	0.086	0.096	0.104
4.75	0.032	0.038	0.044	0.050	0.055	0.061	0.073	0.083	0.093	0.101
5.00	0.031	0.036	0.042	0.048	0.053	0.058	0.070	0.080	0.090	0.098
5.25	0.029	0.035	0.040	0.046	0.051	0.056	0.067	0.078	0.087	0.095
5.50	0.028	0.033	0.039	0.044	0.049	0.054	0.065	0.075	0.084	0.092
5.75	0.027	0.032	0.037	0.042	0.047	0.052	0.063	0.073	0.082	0.090
6.00	0.026	0.031	0.036	0.040	0.045	0.050	0.060	0.070	0.079	0.087
6.25	0.025	0.030	0.034	0.039	0.044	0.048	0.058	0.068	0.077	0.085
6.50	0.024	0.029	0.033	0.038	0.042	0.046	0.056	0.066	0.075	0.083
6.75	0.023	0.028	0.032	0.036	0.041	0.045	0.055	0.064	0.073	0.080
7.00	0.022	0.027	0.031	0.035	0.039	0.043	0.053	0.062	0.071	0.078
7.25	0.022	0.026	0.030	0.034	0.038	0.042	0.051	0.060	0.069	0.076
7.50	0.021	0.025	0.029	0.033	0.037	0.041	0.050	0.059	0.067	0.074
7.75	0.020	0.024	0.028	0.032	0.036	0.039	0.048	0.057	0.065	0.072
8.00	0.020	0.023	0.027	0.031	0.035	0.038	0.047	0.055	0.063	0.071
8.25	0.019	0.023	0.026	0.030	0.034	0.037	0.046	0.054	0.062	0.069
8.50	0.018	0.022	0.026	0.029	0.033	0.036	0.045	0.053	0.060	0.067
8.75	0.018	0.021	0.025	0.028	0.032	0.035	0.043	0.051	0.059	0.066
9.00	0.017	0.021	0.024	0.028	0.031	0.034	0.042	0.050	0.057	0.064
9.25	0.017	0.020	0.024	0.027	0.030	0.033	0.041	0.049	0.056	0.063
9.50	0.017	0.020	0.023	0.026	0.029	0.033	0.040	0.048	0.055	0.061
9.75	0.016	0.019	0.023	0.026	0.029	0.032	0.039	0.047	0.054	0.060
10.00	0.016	0.019	0.022	0.025	0.028	0.031	0.038	0.046	0.052	0.059
20.00	0.008	0.010	0.011	0.013	0.014	0.016	0.020	0.024	0.027	0.031
50.00	0.003	0.004	0.004	0.005	0.006	0.006	0.008	0.010	0.011	0.013
100.00	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.005	0.006	0.006

Table 5.10 - I_f

μs	Df/B	B/L		
		0.2	0.5	1.0
0.3	0.2	0.95	0.93	0.90
	0.4	0.90	0.86	0.81
	0.6	0.85	0.80	0.74
	1.0	0.78	0.71	0.65
0.4	0.2	0.97	0.96	0.93
	0.4	0.93	0.89	0.85
	0.6	0.89	0.84	0.78
	1.0	0.82	0.75	0.69
0.5	0.2	0.99	0.98	0.96
	0.4	0.95	0.93	0.89
	0.6	0.92	0.87	0.82
	1.0	0.85	0.79	0.72

Example



$$S_e := q_o \cdot (\alpha \cdot B) \cdot \frac{(1 - \mu^2)}{E_s} \cdot I_s \cdot I_f$$

$E_s = 10400\text{ kN/m}^2$ (average of all E_s)

$q_o = 150\text{ kN/m}^2$

$B' = B/2 = 1/2 = 0.5\text{ m}$ for center of footing

$u^2 = 0.3^2 = 0.09$

$\alpha = 4$ for center of footing

$m' = L/B = 2$ $n' = H/(B/2) = 5/(1/2) = 10$

$F_1 = 0.641$ $F_2 = 0.031$ from Tables 5.8 & 5.9

$I_s = F_1 + ((1 - 2\mu)/1 - \mu)F_2 = 0.659$

$I_f = 0.71$ from Table 5.10

$S_e = ((150)(4)(0.5)(1 - 0.09)/10400)(0.659)(0.71)$

$S_e = 0.012\text{ m} = 1.2\text{ cm}$

Improved Equation for Elastic Settlement

Mayne & Poulos

- Rigidity
- Depth of embedment
- Increase in E_s with depth
- Location of rigid layers with depth

$$B_e := \sqrt{\frac{(4 BL)}{\pi}} \quad \leftarrow \text{equivalent foundation diameter}$$

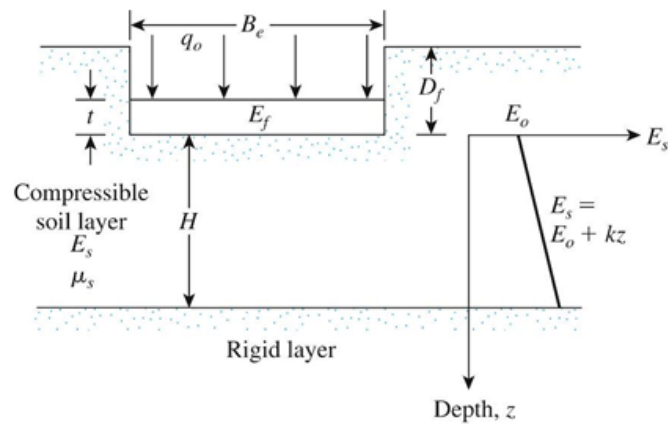
where B = width of foundation
 L = length of foundation

For circular foundations,

$$B_e = B$$

where B = diameter of foundation

Improved equation for calculating elastic settlement: general parameters



$$S_e := \frac{q_o \cdot B_e \cdot I_G \cdot I_F \cdot I_E}{E_o} \cdot 1 - \mu_s^2$$

where

I_G = influence factor for the variation of E_s with depth

I_F = foundation rigidity correction factor

I_E = foundation embedment correction factor

$$I_F = \frac{\pi}{4} + \frac{1}{4.6 + 10 \left(\frac{E_f}{E_o + \frac{B_e}{2} \cdot k} \right) \cdot \left(\frac{2t}{B_e} \right)^3}$$

t = foundation thickness

k = slope of increase in E_s with depth

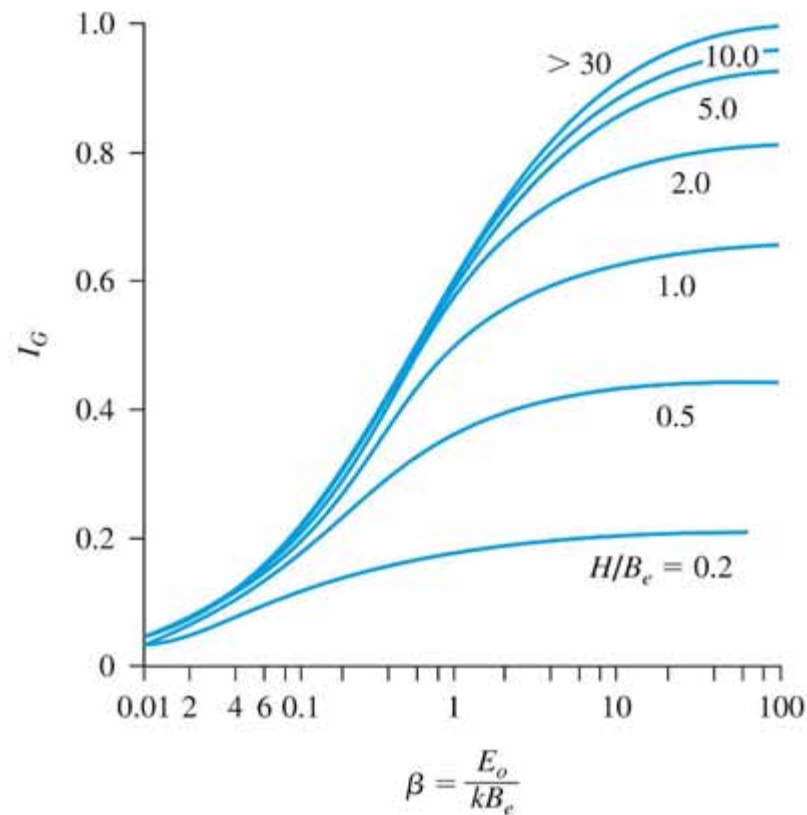
$$E_s = E_o + kz$$

$$E_f = 57000 \sqrt{f'_c}$$

$$E_f = 3,100,000 \text{ psi for 3000 psi concrete}$$

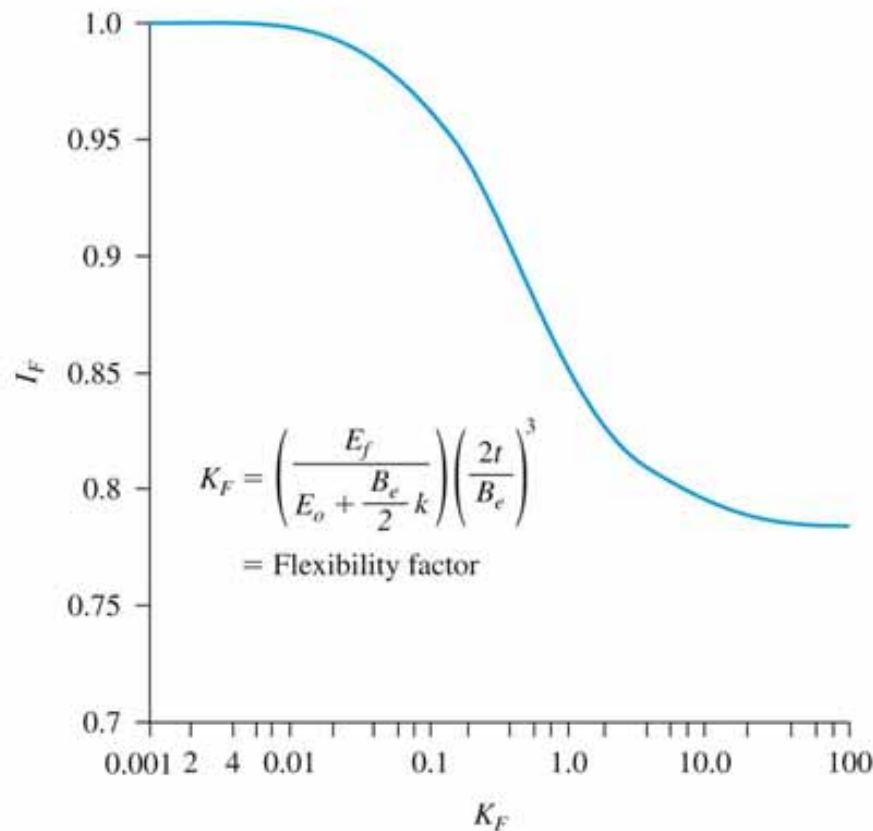
$$I_E := 1 - \frac{1}{3.5 \exp(1.22 \mu_s - 0.4) \cdot \left(\frac{B_e}{D_f} + 1.6 \right)}$$

Variation of I_G with β



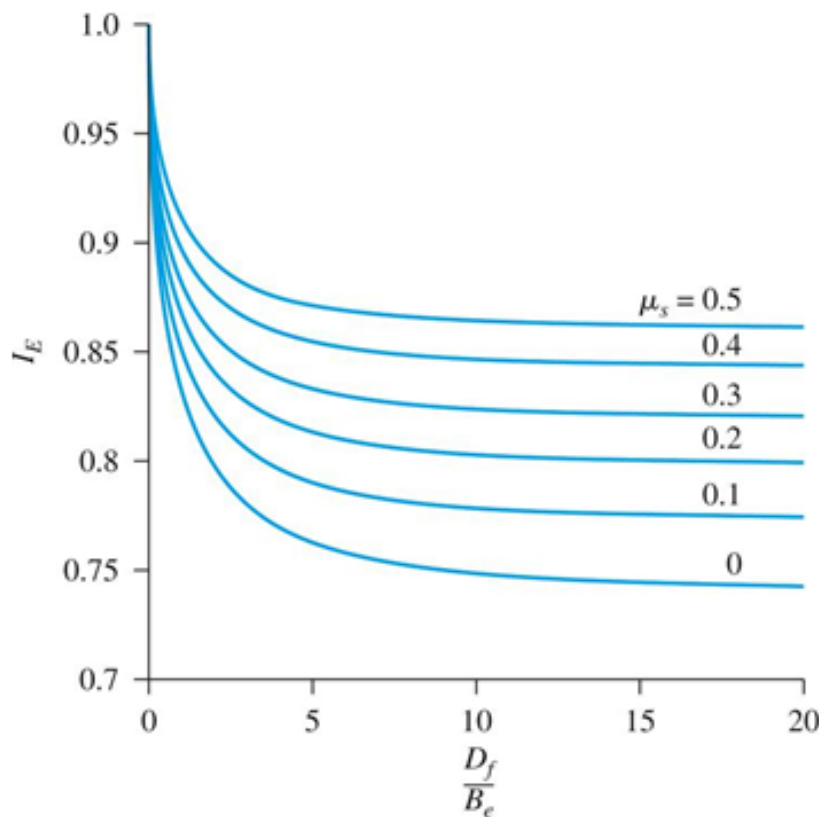
$$\beta := \frac{E_o}{kB_e}$$

Variation of rigidity correction factor I_F with flexibility factor K_F



$$I_F := \frac{\pi}{4} + \frac{1}{4.6 + 10 \left(\frac{E_f}{E_o + \frac{B_e}{2}k} \right) \left(\frac{2t}{B_e} \right)^3}$$

Variation of embedment correction factor I_E with D_f/B_e



$$I_E := 1 - \frac{1}{3.5 \exp(1.22 \mu_s - 0.4) \left(\frac{B_e}{D_f} + 1.6 \right)}$$

Example 5.6 in book

Schmertmann Strain Influence Factor

$$S_e := C_1 \cdot C_2 \cdot (\bar{q} - q) \cdot \sum_{Z=0}^{Z_2} \left(\frac{I_z}{E_s} \cdot \Delta Z \right)$$

where

- I_z = strain influence factor
- C_1 = foundation embedment correction factor = $1 - 0.5[q/(\bar{q} - q)]$
- C_2 = creep correction factor = $1 + 0.2 \cdot \log(\text{time in years}/0.1)$
- \bar{q} = stress at level of foundation
- $q = \gamma D_f$

Strain influence factor variation dependent on foundation shape.

For square or circular foundations:

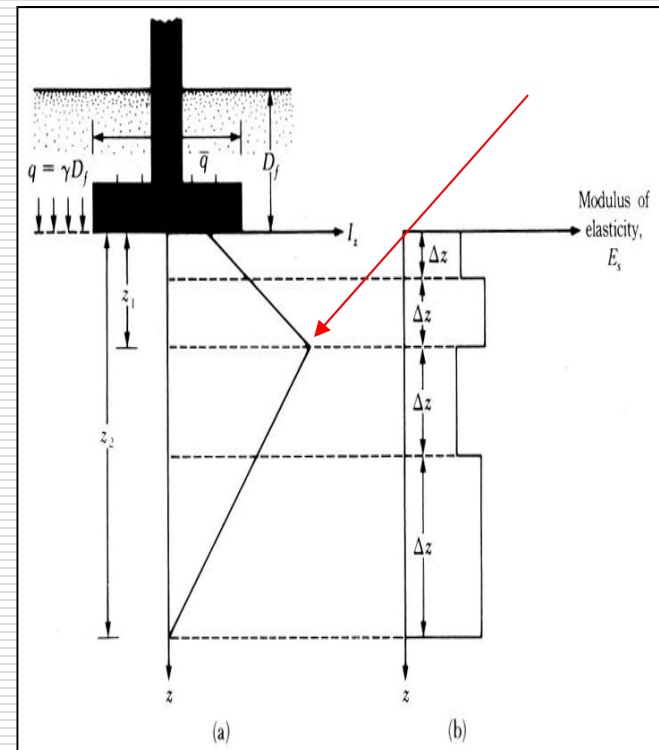
$$\begin{aligned} I_z &= 0.1 & \text{at } z=0 \\ I_z &= 0.5 & \text{at } z_1 = 0.5B \\ I_z &= 0 & \text{at } z_2 = 2B \end{aligned}$$

For continuous foundations ($L/B > 10$):

$$\begin{aligned} I_z &= 0.2 & \text{at } z=0 \\ I_z &= 0.5 & \text{at } z_1 = B \\ I_z &= 0 & \text{at } z_2 = 4B \end{aligned}$$

Or use equations 5.51
thru 5.53
for rectangular footing

Values of L/B between 1 and 10 can be interpolated.



Note the division of the soil based on type & I_z

Example 5.7 in Book

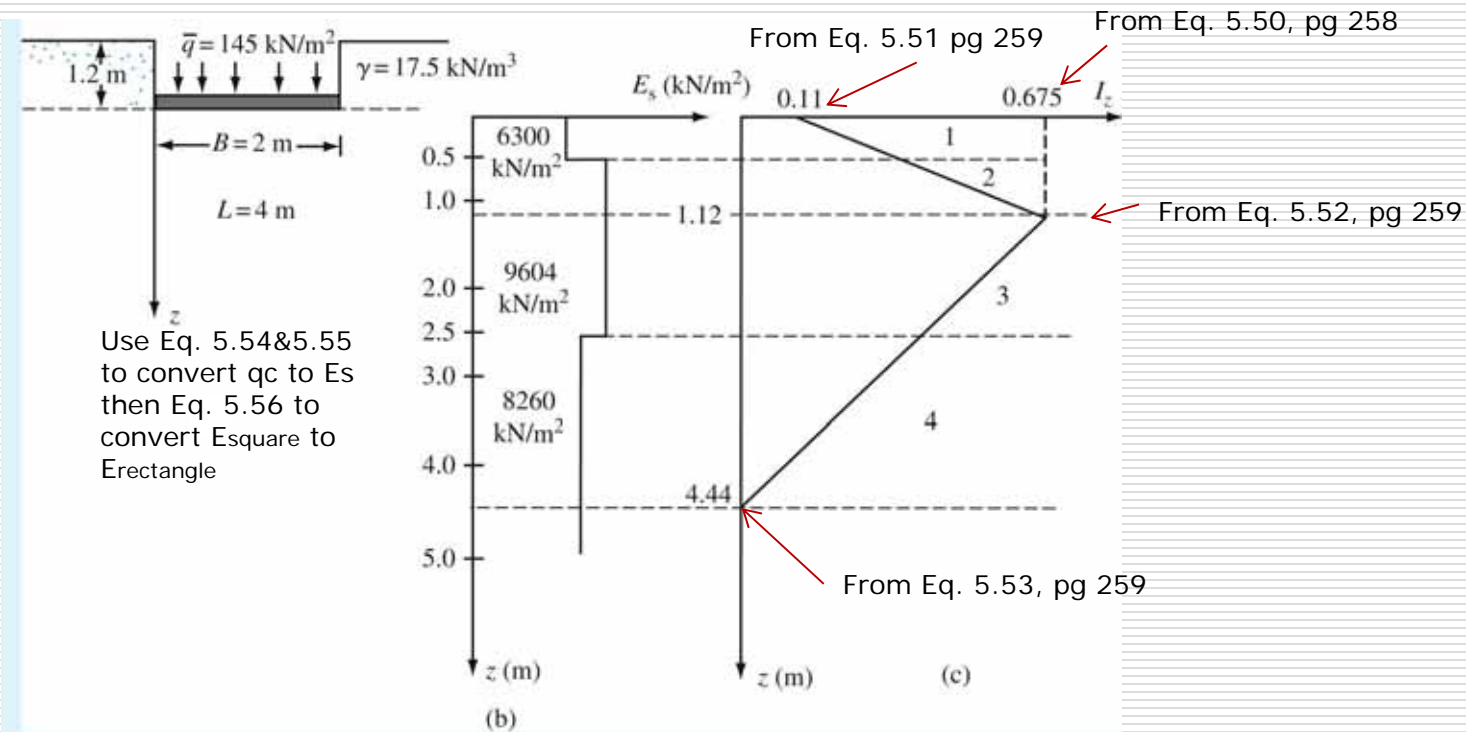


Figure 5.23

Example 5.7 Continued

Layer no.	Δz (m)	E_s (kN/m ²)	I_z at middle of layer	$\frac{I_z}{E_s} \Delta z$ (m ³ /kN)
1	0.50	6300	0.236	1.87×10^{-5}
2	0.62	9604	0.519	3.35×10^{-5}
3	1.38	9604	0.535	7.68×10^{-5}
4	1.94	8260	0.197	4.62×10^{-5}
				$\Sigma 17.52 \times 10^{-5}$

$$S_e = C_1 C_2 (\bar{q} - q) \sum \frac{I_z}{E_s} \Delta z$$

$$C_1 = 1 - 0.5 \left(\frac{q}{\bar{q} - q} \right) = 1 - 0.5 \left(\frac{21}{145 - 21} \right) = 0.915$$

Assume the time for creep is 10 years. So,

$$C_2 = 1 + 0.2 \log \left(\frac{10}{0.1} \right) = 1.4$$

Hence,

$$S_e = (0.915)(1.4)(145 - 21)(17.52 \times 10^{-5}) = 2783 \times 10^{-5} \text{ m} = \mathbf{27.83 \text{ mm}}$$

To find I_z get the slope of the line. For the top line it's $(0.675 - 0.11)/1.12 = 0.504$

Half way in Layer 1 is 0.25m
so $0.25 * 0.504 = 0.126$

Add 0.11 to 0.126 and get 0.236

Once you get on the second line, find it's slope and repeat.

Stress Increase Under Embankment

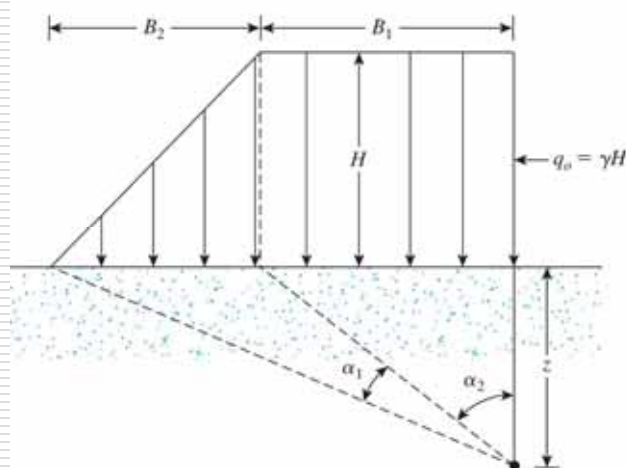


Figure 5.10 Embankment loading

$$\Delta\sigma := \frac{q_0}{\pi} \cdot \left[\left(\frac{B_1 + B_2}{B_2} \right) \cdot (\alpha_1 + \alpha_2) - \frac{B_1}{B_2} \cdot \alpha_2 \right]$$

$$\alpha_1 := \tan^{-1} \left[\frac{(B_1 + B_2)}{z} \right] - \tan^{-1} \left(\frac{B_1}{z} \right)$$

$$\alpha_2 := \tan^{-1} \left(\frac{B_1}{z} \right)$$

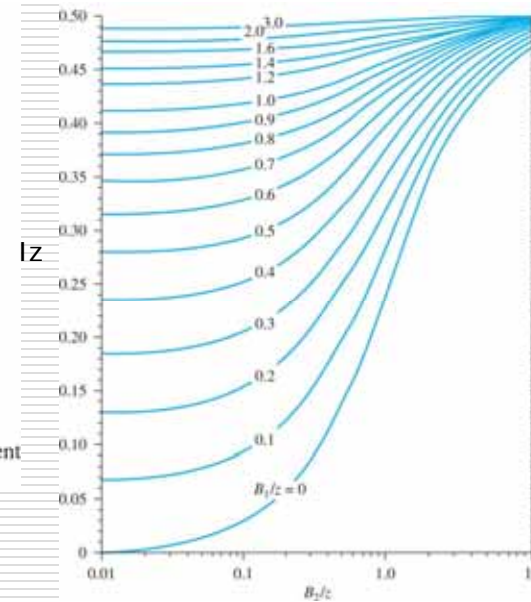


Figure 5.11 Influence value I' for embankment loading (After Osterberg, 1957) (Osterberg, J. O. (1957). "Influence Values for Vertical Stresses in Semi-Infinite Mass Due to Embankment Loading," Proceedings, Fourth International Conference on Soil Mechanics and Foundation Engineering, London, Vol. 1, pp. 393-396. With permission from ASCE.)

$$\Delta\sigma := q_0 \cdot I$$

Will need this on 430 projects

Example 5.3

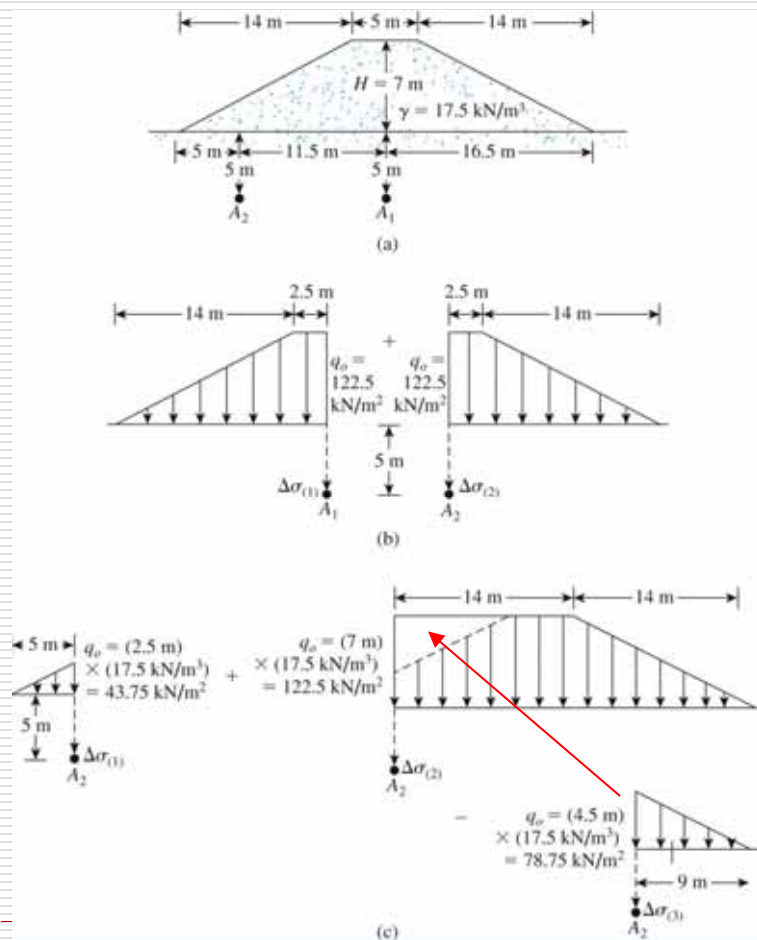


Figure 5.12 Stress increase due to embankment loading

Problem in (a) and (b)

$$\frac{B_1}{z} := \frac{2.5}{5} = 0.5 \quad \frac{B_2}{z} := \frac{14}{5} = 2.8 \quad I' = 0.445$$

$$\Delta\sigma := \Delta\sigma_1 + \Delta\sigma_2 \quad \Delta\sigma := q_o \cdot (I'_{\text{leftside}} + I'_{\text{rightside}})$$

$$\Delta\sigma := 122.5(0.445 + 0.445) = 109 \text{ kN/m}^2$$

Problem in (c)

$$\Delta\sigma := \Delta\sigma_1 + \Delta\sigma_2 - \Delta\sigma_3$$

Material Parameters

From Properties Table

Table 5.6 Elastic Parameters of Various Soils

Type of soil	Modulus of elasticity, E_s		Poisson's ratio, μ_s
	MN/m ²	lb/in ²	
Loose sand	10.5–24.0	1500–3500	0.20–0.40
Medium dense sand	17.25–27.60	2500–4000	0.25–0.40
Dense sand	34.50–55.20	5000–8000	0.30–0.45
Silty sand	10.35–17.25	1500–2500	0.20–0.40
Sand and gravel	69.00–172.50	10,000–25,000	0.15–0.35
Soft clay	4.1–20.7	600–3000	0.20–0.50
Medium clay	20.7–41.4	3000–6000	
Stiff clay	41.4–96.6	6000–14,000	

Will need this page on midterm exam.

From CPT Data

$E_s = 2.5q_c$ (square & circular)

$E_s = 3.5q_c$ for continuous

From SPT Data

$$\frac{E_s}{p_a} := 8 \cdot N_{60}$$

where N_{60} = corrected SPT value

p_a = atmospheric pressure = 1 tsf

From Lab Data

Normally consolidated clays

$E_s = 250c_u$ to $500c_u$

Overconsolidated clays

$E_s = 750c_u$ to $1000c_u$

Seismic Bearing Capacity

$$q_u = qN_q + \frac{1}{2}\gamma BN_\gamma \text{ (static conditions)}$$

$$q_{uE} = qN_{qE} + \frac{1}{2}\gamma BN_{\gamma E} \text{ (earthquake conditions)}$$

where N_q , N_γ , N_{qE} , $N_{\gamma E}$ = bearing capacity factors
 $q = \gamma D_f$

Note:

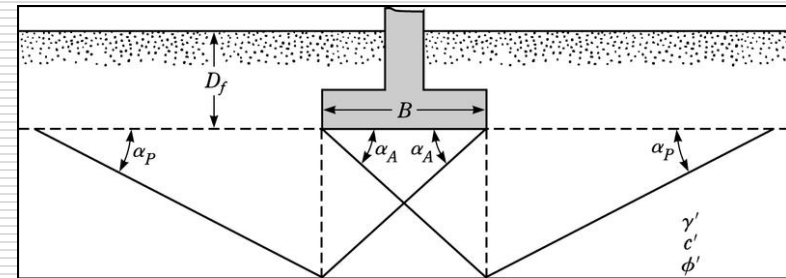
$$N_q \text{ and } N_\gamma = f(\phi')$$

and

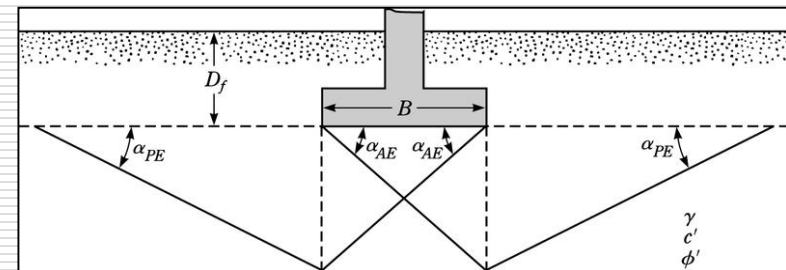
$$N_{qE} \text{ and } N_{\gamma E} = f(\phi', \tan \theta)$$

where $\tan \theta = k_h / (1 - k_v)$

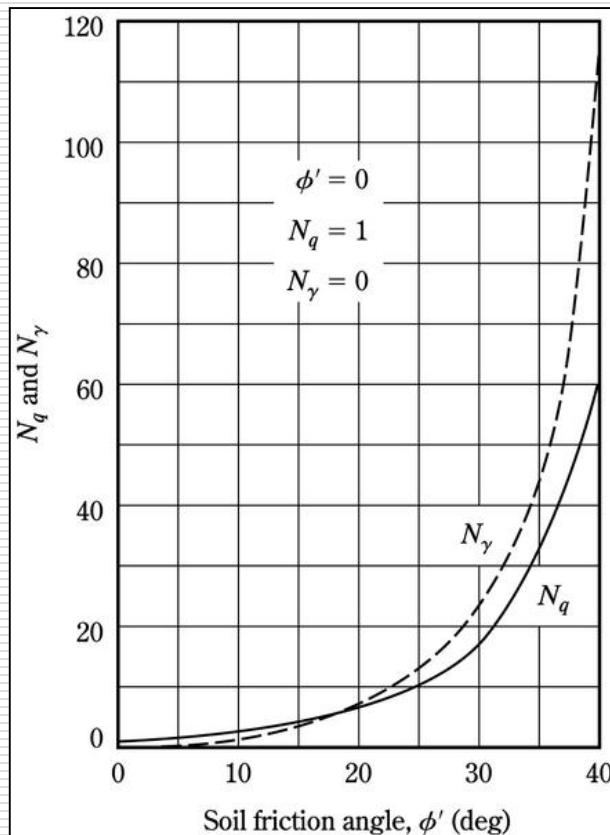
with k_h = horizontal coefficient of acceleration
 k_v = vertical coefficient of acceleration



(Note: $\alpha_A = 45 + \phi'/2$ and $\alpha_P = 45 - \phi'/2$)



Variation of N_q and N_γ



Based on
simplified
failure surface

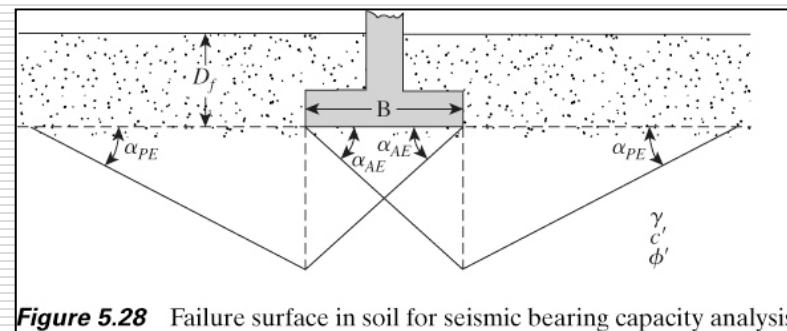


Figure 5.28 Failure surface in soil for seismic bearing capacity analysis

(Note: $\alpha_{AE} = 45 + \phi'/2$ and $\alpha_{PE} = 45 - \phi'/2$)

Variation of $N_{\gamma E}/N_{\gamma}$ and N_{qE}/N_q

(after Richards et al., 1993)

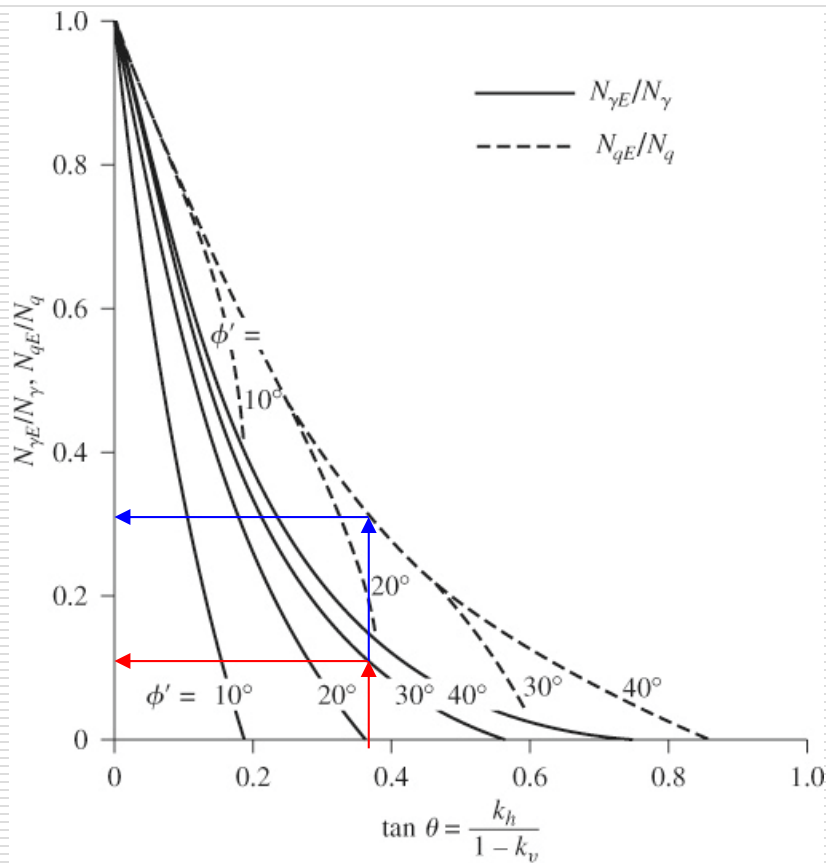
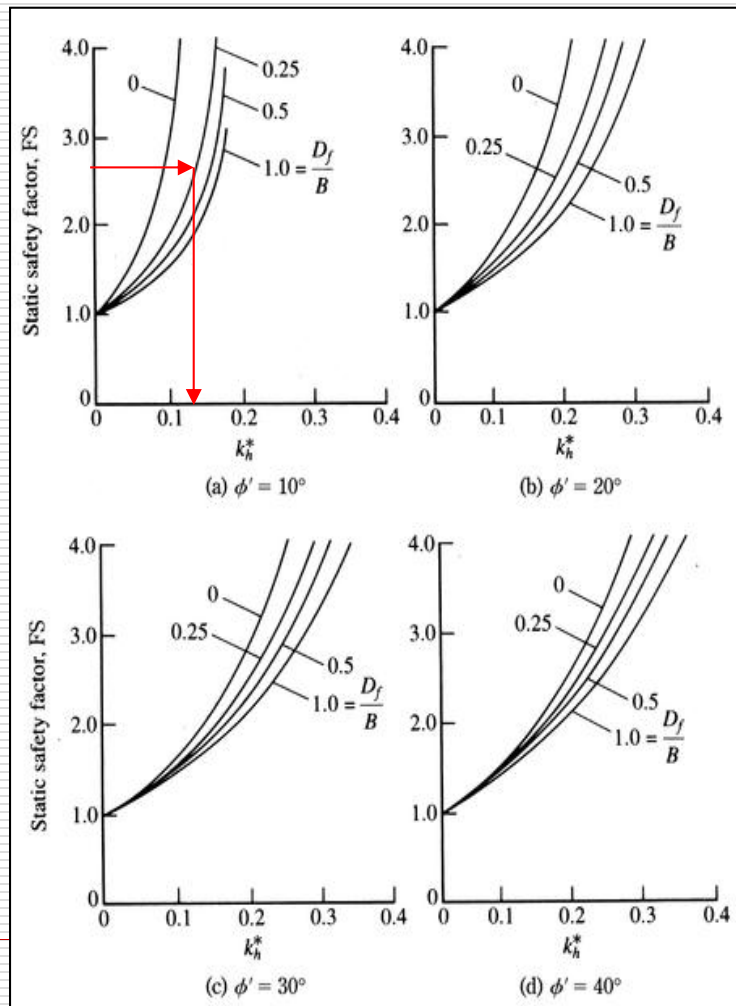


Figure 5.30 Variation of $N_{\gamma E}/N_{\gamma}$ and N_{qE}/N_q with $\tan \theta$

Critical acceleration k_h^* for $c' = 0$



Seismic Settlement

$$S_{Eq} := 0.174 \frac{V^2}{Ag} \cdot \left(\frac{k_h^*}{A} \right)^{-4} \cdot \tan(\alpha_{AE}) \quad (\text{in meters})$$

V = peak velocity for the design earthquake (m/sec)

A = acceleration coefficient for the design earthquake

g = acceleration due to gravity (9.18 m/sec²)

α_{AE}

Table 5.11 Variation of $\tan \alpha_{AE}$ with k_h^* and soil friction angle ϕ'
(Compiled from Richards et al., 1993)

k_h^*	$\tan \alpha_{AE}$				
	$\phi' = 20^\circ$	$\phi' = 25^\circ$	$\phi' = 30^\circ$	$\phi' = 35^\circ$	$\phi' = 40^\circ$
0.05	1.10	1.24	1.39	1.57	1.75
0.10	0.97	1.13	1.26	1.44	1.63
0.15	0.82	1.00	1.15	1.32	1.48
0.20	0.71	0.87	1.02	1.18	1.35
0.25	0.56	0.74	0.92	1.06	1.23
0.30		0.61	0.77	0.94	1.10
0.35		0.47	0.66	0.84	0.98
0.40		0.32	0.55	0.73	0.88
0.45			0.42	0.63	0.79
0.50			0.27	0.50	0.68
0.55				0.44	0.60
0.60				0.32	0.50

Seismic Example

A strip foundation is to be constructed on a sandy soil with $B=4\text{ft}$, $D_f=3\text{ft}$, $\gamma=110\text{ lb/ft}^3$ and $\phi = 30^\circ$.

- Determine the gross ultimate bearing capacity q_{uE} . Assume $k_v=0$ and $k_h=0.176$.
- If the design earthquake parameters are $V = 1.3\text{ ft/sec}$ and $A=9.81\text{m/sec}^2$, determine the seismic settlement of the foundation. Use $FS=3$ to obtain the static allowable bearing capacity.

Solution

Part a

From Fig 5.29, for $\phi = 30^\circ$, $N_q = 16.51$ and $N_\gamma = 23.76$. Also

$$\tan \theta = k_h/(1-k_v) = 0.176$$

For $\tan \theta = 0.176$, Figure 5.30 gives

$$N_{\gamma E}/N_\gamma = 0.4 \text{ and } N_{qE}/N_q = 0.63$$

Thus,

$$N_{\gamma E} = (0.4)(23.76) = 9.5$$

$$N_{qE} = (0.63)(16.51) = 10.4$$

Seismic Example (cont)

And

$$q_{uE} = qN_{qE} + 0.5\gamma B N_{\gamma E} = (3)(110)(10.4) + (0.5)(110)(4)(9.5) = 5522 \text{ lb/ft}^2$$

Part b

For the foundation, $D_f/B = 3/4 = 0.75$

From Figure 5.31 for $\phi = 30^\circ$, $FS = 3$, and $D_f/B = 0.75$, the value $k_h^* = 0.26$

Also, from Table 5.11, for $k_h^* = 0.26$, the value of $\tan \alpha_{AE} = 0.92$.

$$S_{Eq} = 0.174(k_h^*/A)^{-4} \tan \alpha_{AE}(V^2/Ag) \text{ (meters)}$$

With

$$V = 1.3 \text{ ft} = 0.4 \text{ m}$$

Then

$$S_{Eq} = 0.174(0.4)^2 / ((0.32)(9.81)) (0.26/0.32)^{-4} (0.92) = 0.0187 \text{ m} = 0.74 \text{ in}$$

Primary Consolidation

$$\frac{\Delta H}{H} := \frac{\Delta e}{1 + e_o} \quad \Delta e := C_c \cdot \log \left(\frac{\sigma_{o+\Delta\sigma}}{\sigma_o} \right)$$

$$S_{c(p)} = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o} \quad (\text{for normally consolidated clays}) \quad (1.53)$$

$$S_{c(p)} = \frac{C_s H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o} \quad (\text{for overconsolidated clays with } \sigma'_o + \Delta\sigma'_{av} < \sigma'_c) \quad (1.55)$$

$$S_{c(p)} = \frac{C_s H_c}{1 + e_o} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_c} \quad (\text{for overconsolidated clays with } \sigma'_o < \sigma'_c < \sigma'_o + \Delta\sigma'_{av}) \quad (1.57)$$

where σ'_o = average effective pressure on the clay layer before the construction of the foundation

$\Delta\sigma'_{av}$ = average increase in effective pressure on the clay layer caused by the construction of the foundation

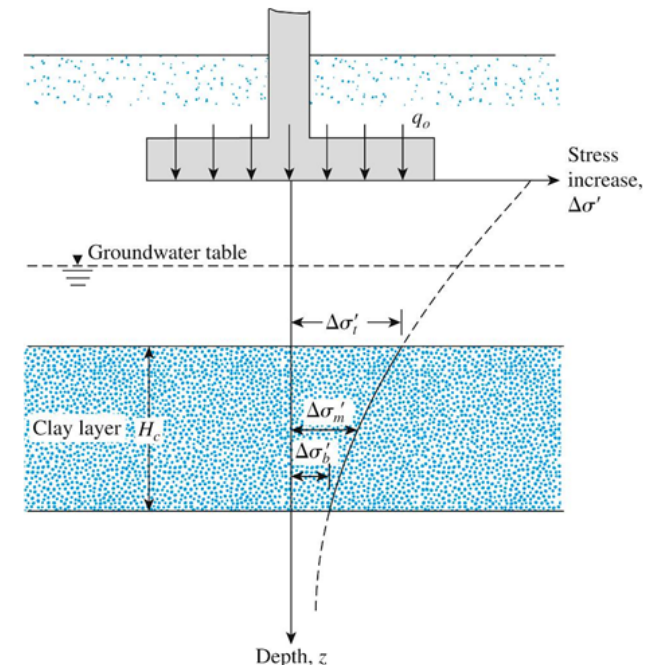
σ'_c = preconsolidation pressure

e_o = initial void ratio of the clay layer

C_c = compression index

C_s = swelling index

H_c = thickness of the clay layer



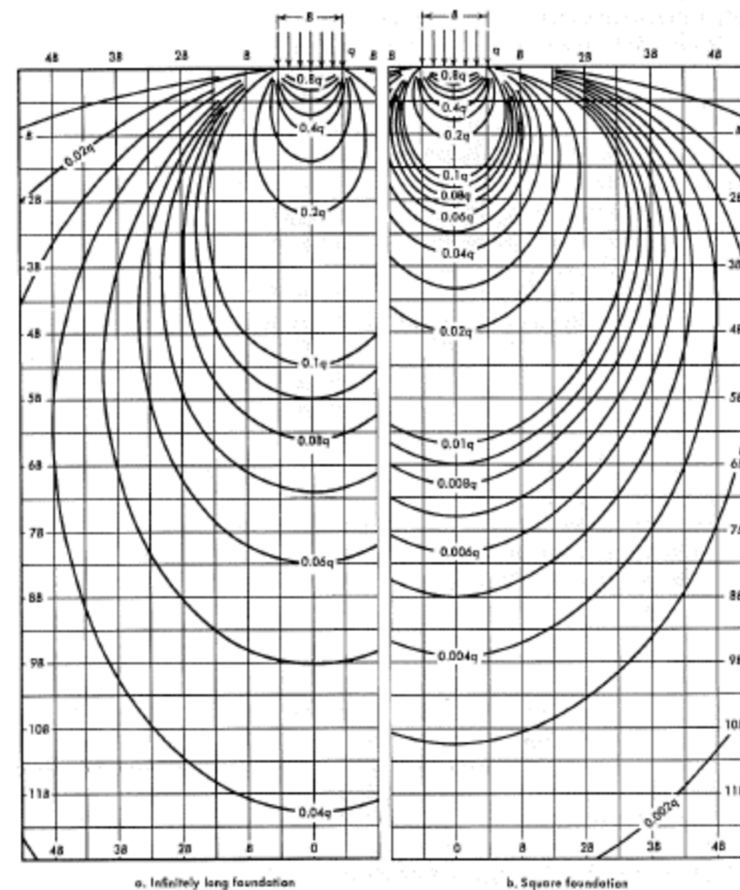
$$\Delta\sigma'_{avg} := \frac{1}{6} (\Delta\sigma'_t + 4 \cdot \Delta\sigma'_m + \Delta\sigma'_b)$$

Westergaard

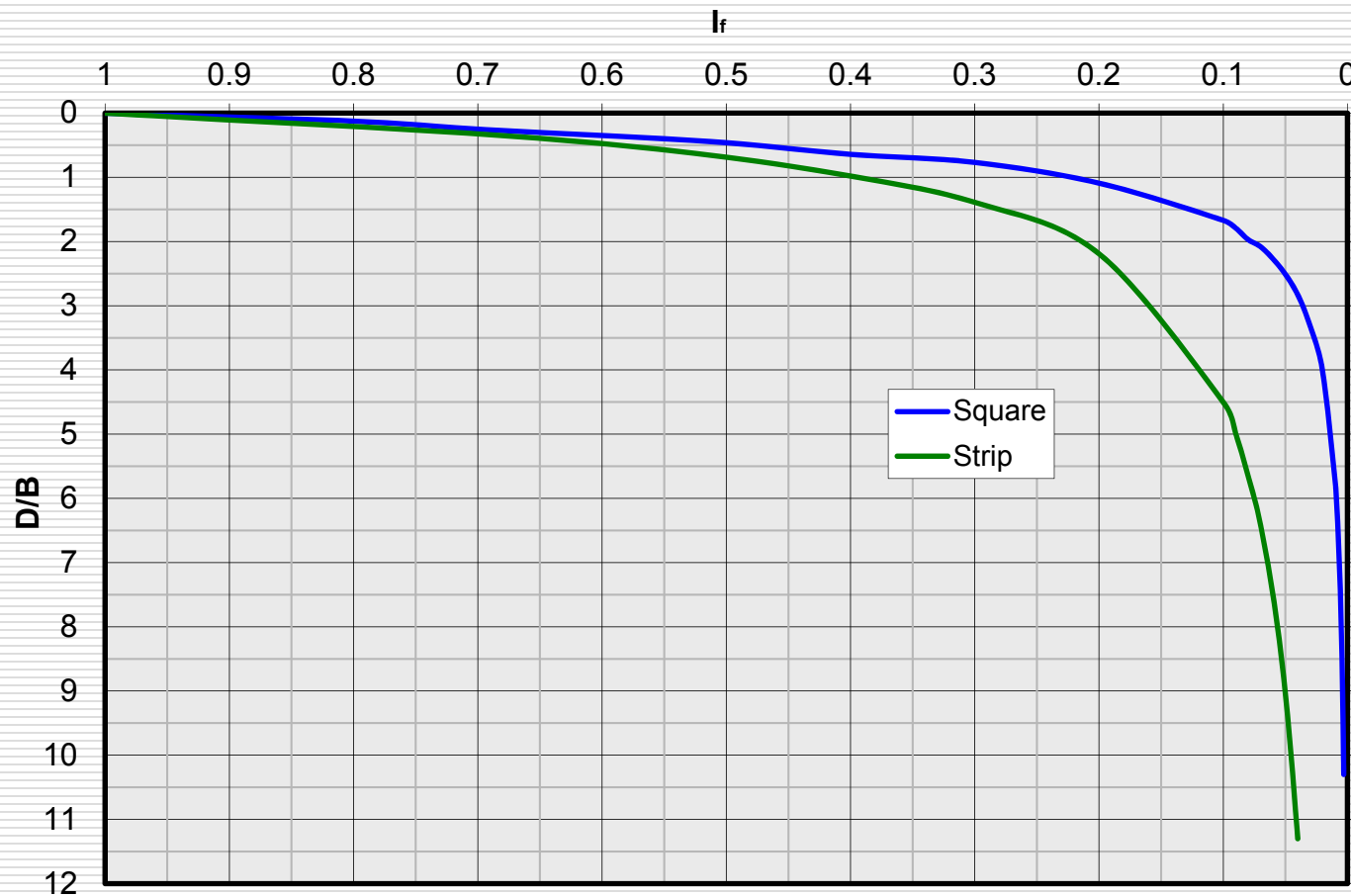
Another method for determining the increase in stress within a soil layer with depth is using the Westergaard or Boussinesq stress distribution.

Westergaard assumes a layered subsurface while Boussinesq assumes a homogenous throughout the subsurface.

$\Delta\sigma$ is determined from charts and used in same equations
Charts are in units of B .



Westergaard Centerline Δq



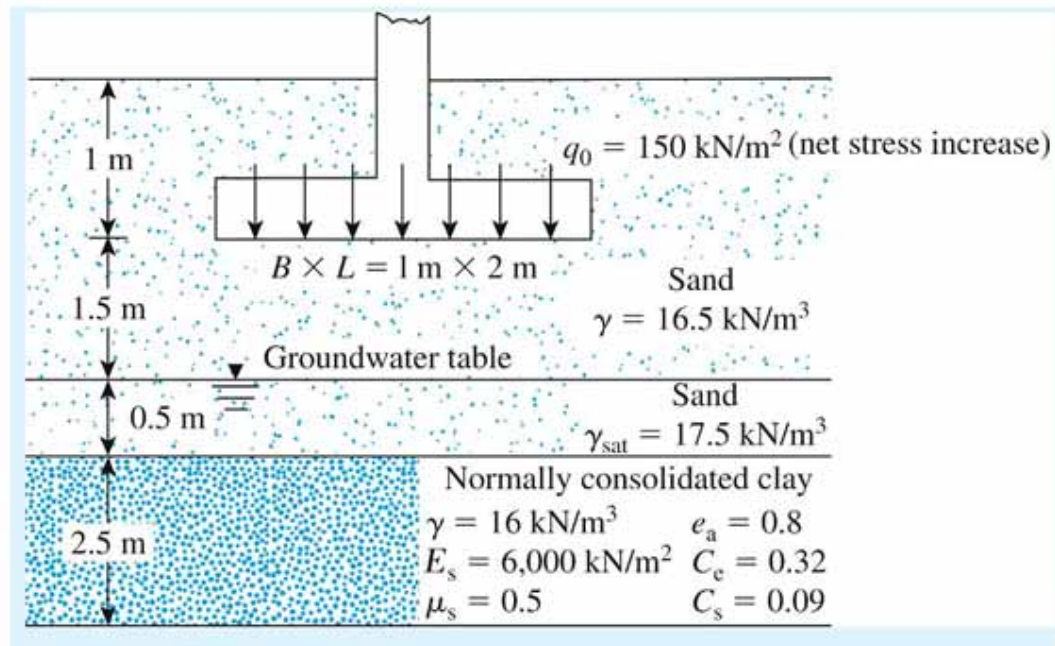
$$\Delta q = I_f(q_o)$$

Using Westergaard

The Westergaard chart is in units of B , both in the Z -direction as well as the X -direction. For the center of a foundation $X/B = 0$ and you use the Westergaard Centerline Δq chart for convenience. For the depth where you want to determine Δq , you divide the depth by the footing width (D/B). Find D/B on the left axis, draw a line straight over to either the strip or square footing line, then draw a line straight up to get I_f . Multiply the footing pressure by I_f and get the Δq for that depth.

For determining what pressure a footing exerts on a nearby underground structure, use the main chart. Determine X/B where X is the lateral distance to the object, then determine Z/B where Z is the depth to the object. Read I_f from the chart, then multiply by footing pressure to get the pressure on the object.

Westergaard Problem



The clay is normally consolidated so we use the following equation:

$$S_c := \frac{(C_c \cdot H_c)}{1 + e_o} \cdot \log \left(\frac{\sigma'_o + \Delta \sigma'_{\text{avg}}}{\sigma'_o} \right)$$

Westergaard Cont.

Turn rectangular footing into equivalent square footing $-\sqrt{2 \cdot 1} = 1.414\text{m}$ Use 1.4m

$$\sigma'_o = (2.5)(16.5) + (0.5)(17.5 - 9.81) + (1.25)(16 - 9.81) = 52.84 \text{ kN/m}^2$$

Next, determine average change in pressure in the clay layer.

$$\Delta\sigma_{\text{avg}} := \frac{1}{6} \cdot (\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b)$$

Depth (m)	D/B	I_f	$\Delta\sigma$
2.00	1.43	0.13	$0.13(150) = 19.5$
3.25	2.32	0.06	$0.06(150) = 9.0$
4.50	3.21	0.03	$0.03(150) = 4.5$

$$\Delta\sigma_{\text{avg}} := \frac{1}{6} \cdot [19.5 + (4) \cdot (9) + 4.5] = 10 \text{ kN/m}^2$$

$$S_c := \frac{(0.32 \cdot 2.5)}{1 + 0.8} \cdot \log \left[\frac{(52.84 + 10)}{52.84} \right] = 0.033\text{m or } 33\text{mm}$$

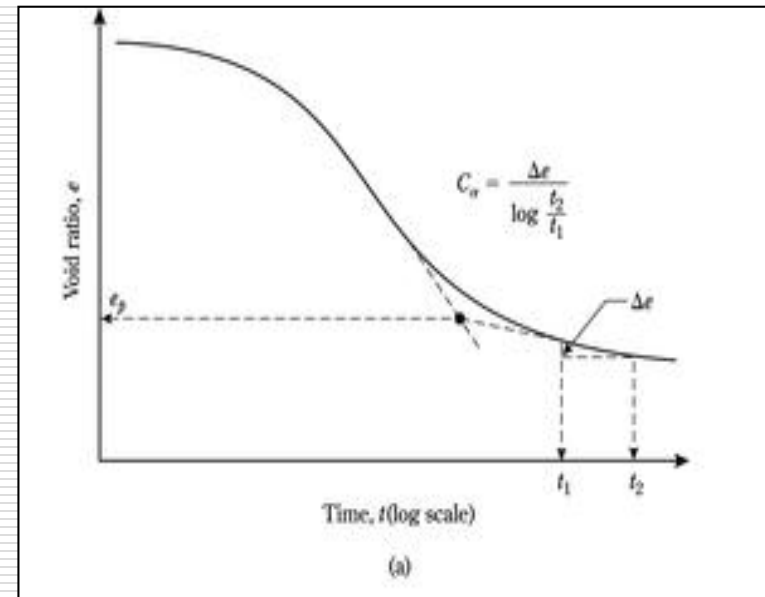
Secondary Consolidation

$$C_a := \frac{\Delta e}{\log\left(\frac{t_2}{t_1}\right)}$$

where C_a = secondary compression index
 Δe = change in void ratio
 t_1, t_2 = time

$$S_{c(s)} := \frac{C_a}{1 + e_p} \cdot H_c \cdot \log\left(\frac{t_2}{t_1}\right)$$

e_p = void ratio at the end of primary consolidation
 H_c = thickness of clay layer



C_α Empirical Correlations

$C_\alpha = 0.0001w$ for overconsolidated soils

$C_\alpha / C_c = 0.04$ for inorganic clays and silts

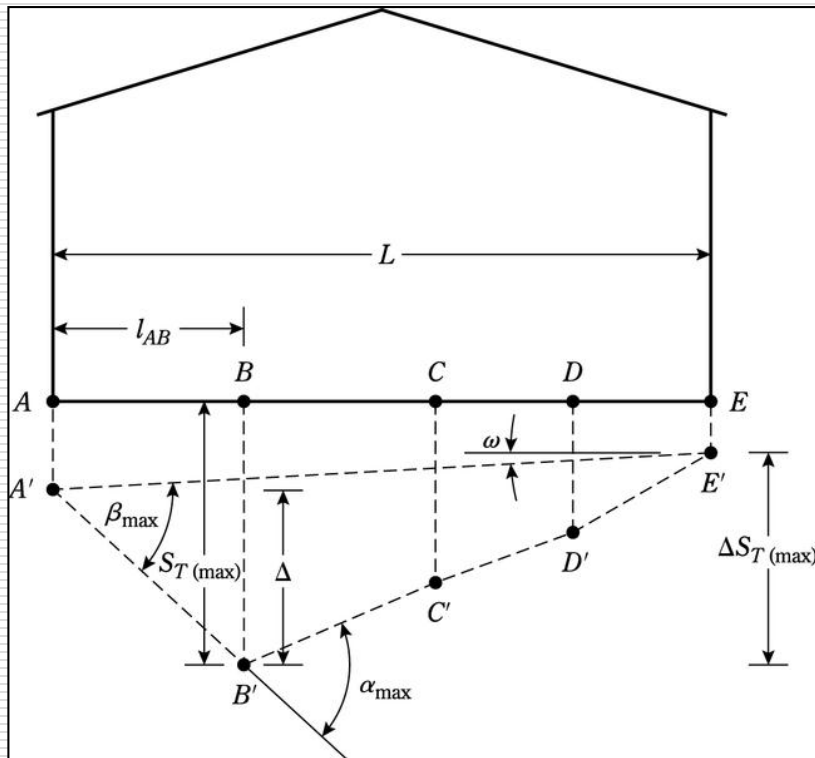
$C_\alpha / C_c = 0.05$ for organic clays and silts

$C_\alpha / C_c = 0.075$ for peats

Tolerable Settlements of Buildings

Two Settlements of Concern

- Total Settlement
- Differential Settlement



S_T = total settlement at a given point

ΔS_T = difference in settlement between any 2 points, also called differential settlement

α = gradient between any 2 successive points

β = angular distortion = $\Delta S_T / l$

ω = tilt

Δ = relative deflection

Δ / L = deflection ratio

Limiting Values of Settlement

In 1956, Skempton and McDonald proposed the following limiting values for maximum settlement and maximum angular distortion, to be used for building purposes:

Maximum settlement, $S_{T(max)}$	
In sand	32 mm
In clay	45 mm
Maximum differential settlement, $\Delta S_{T(max)}$	
Isolated foundations in sand	51 mm
Isolated foundations in clay	76 mm
Raft in sand	51–76 mm
Raft in clay	76–127 mm
Maximum angular distortion, β_{max}	1/300

On the basis of experience, Polshin and Tokar (1957) suggested the following allowable deflection ratios for buildings as a function of L/H , the ratio of the length to the height of a building:

$$\Delta/L = 0.0003 \text{ for } L/H \leq 2$$

$$\Delta/L = 0.001 \text{ for } L/H = 8$$

The 1955 Soviet Code of Practice gives the following allowable values:

Type of building	L/H	Δ/L
Multistory buildings and civil dwellings	≤ 3	0.0003 (for sand)
		0.0004 (for clay)
	≥ 5	0.0005 (for sand)
		0.0007 (for clay)
One-story mills		0.001 (for sand and clay)

Bjerrum (1963) recommended the following limiting angular distortion, β_{max}

Category of potential damage	β_{max}
Safe limit for flexible brick wall ($L/H > 4$)	1/150
Danger of structural damage to most buildings	1/150
Cracking of panel and brick walls	1/150
Visible tilting of high rigid buildings	1/250
First cracking of panel walls	1/300
Safe limit for no cracking of building	1/500
Danger to frames with diagonals	1/600

Table 5.8 Recommendations of European Committee for Standardization on Differential Settlement Parameters

Item	Parameter	Magnitude	Comments
Limiting values for serviceability (European Committee for Standardization, 1994a)	S_T	25 mm	Isolated shallow foundation
		50 mm	Raft foundation
	ΔS_T	5 mm	Frames with rigid cladding
		10 mm	Frames with flexible cladding
		20 mm	Open frames
	β	1/500	—
Maximum acceptable foundation movement (European Committee for Standardization, 1994b)	S_T	50	Isolated shallow foundation
	ΔS_T	20	Isolated shallow foundation
	β	$\approx 1/500$	—

Locally – 1 inch maximum for columns, ¾ inch maximum for walls

Homework

From Chapter 5

CE 430

- ☐ 5.7
- ☐ 5.11
- ☐ 5.13
- ☐ 5.20 Using Westergaard
- ☐ 5.5 but using Westergaard

CE 530

Same as CE430

Next Week
Hand Out Projects